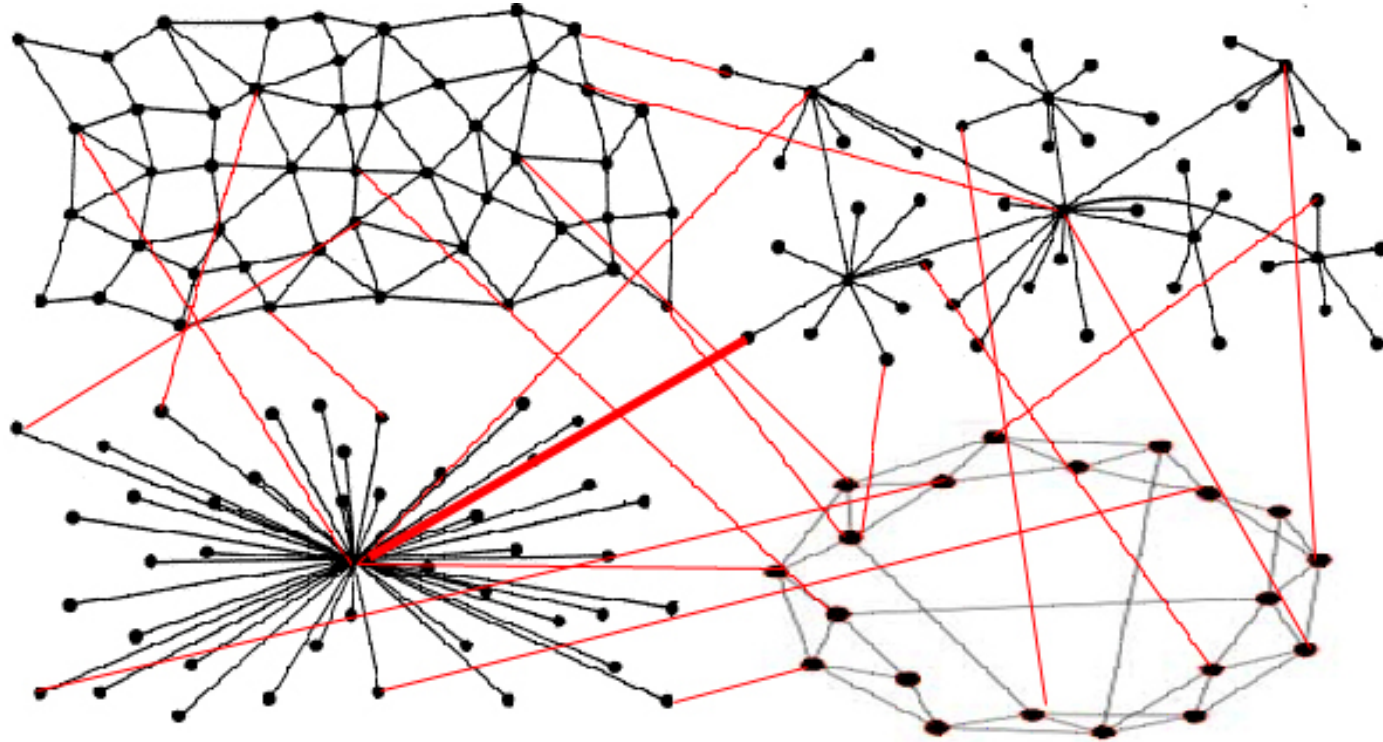


# The Extreme Vulnerability of Network of Networks



Protein networks,  
Brain networks  
Physiological systems  
Infrastructures

Shlomo Havlin  
Bar-Ilan University  
Israel

Two types of **links**:  
1. Connectivity  
2. **Dependency**

**Cascading disaster-Sudden collapse**  
.....

MULTIPLEX IS A SPECIAL CASE OF **NON**

# From Single Network to Network of Networks

## Collaboration:

Amir Bashan, BIU

Yehiel Berezin, BIU

Sergey Buldyrev, NY

Jianxi Gao: Northeastern

Hans Herrmann, Zurich

Stefano Boccalleti, Florence

Jose Andrade, Fortaleza

Wei Li, BU

Hernan Makse, NY

Xuqing Huang, BU

Roni Parshani: BIU

Nuno Araujo, ETH

Javier M Buldu, Madrid

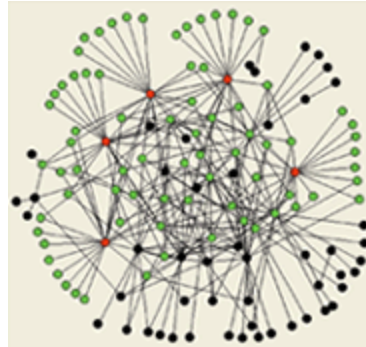
Irene Sendina–Nadal, Madrid

Christian Schneider, MIT

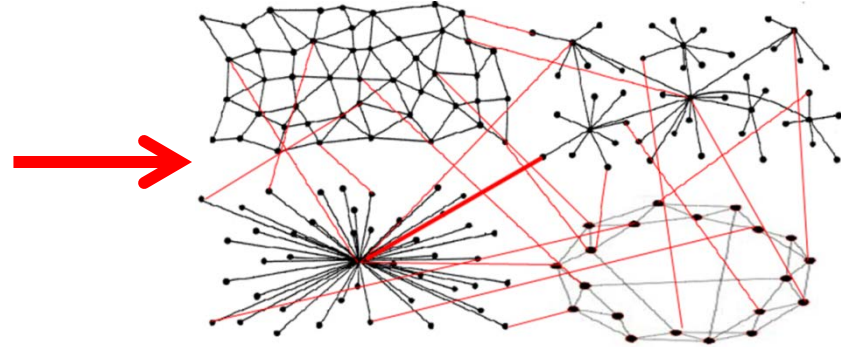
H. E. Stanley, Boston

PARTIAL LIST

2000



2010



Buldyrev et al, Nature, 464, 1025 (2010 )

Parshani et al, PRL ,105, 0484 (2010)

Schneider et PNAS, 108, 3838 (2010)

Parshani et al, PNAS, 108, 1007 (2011)

Gao et al, PRL, 107, 195701 (2011)

Gao et al, Nature Phys.,8, 40 (2012)

Bashan et al, Nature Com., 3, 702, (2012)

Wei Li et al, PRL, 108, 228702 (2012)

Bashan et al, arXiv:1206.2062 (Nature Phys. (2013))

Schneider et al, Scientific Reports (2013)

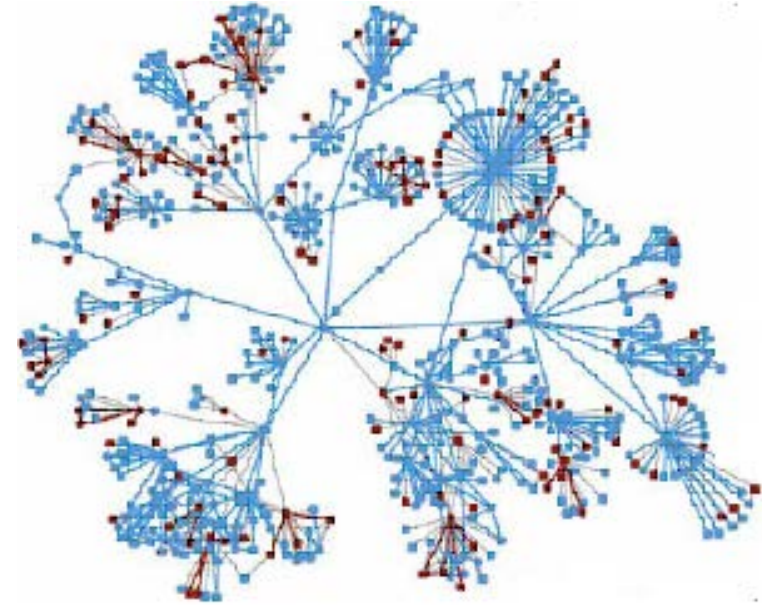
# Extensive Studies Since 2000 -- Single Networks

- A **Network** is a structure of  $N$  **nodes** and  $M$  **edges** (or  $2M$  **links** )
- Called usually **graph** – in Mathematics
- Complex systems can be described and understood using networks

**Internet:** nodes represent computers  
links the connecting cables

**Biological systems:** nodes represent  
proteins links their relations

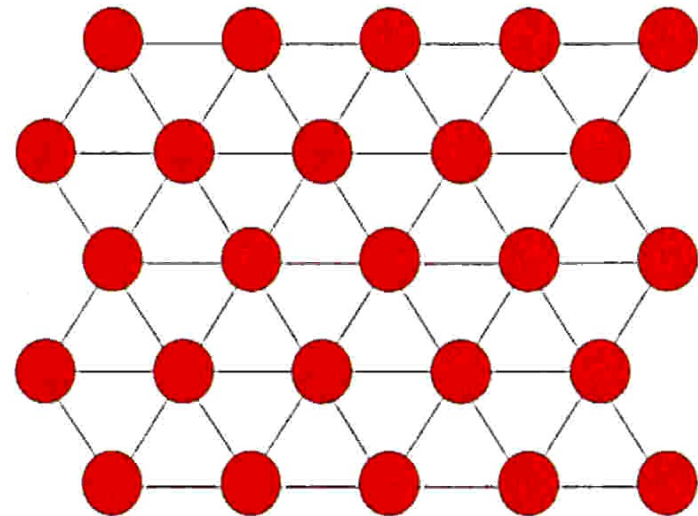
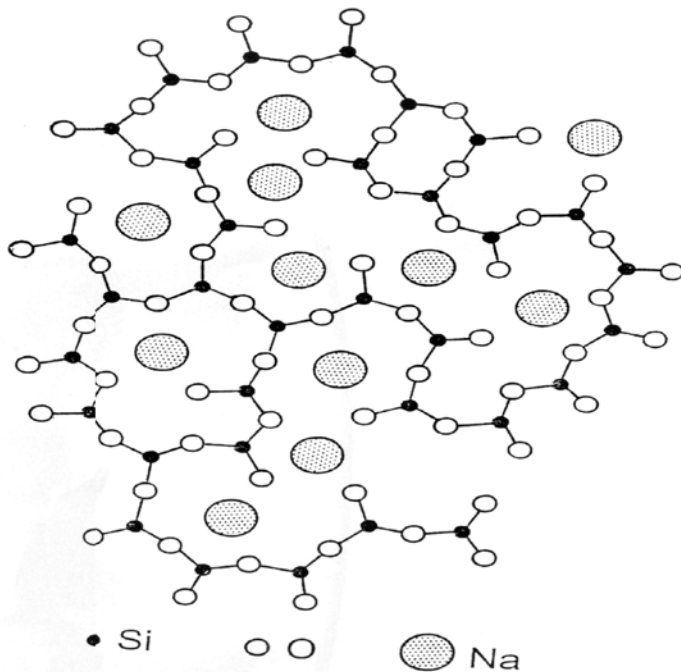
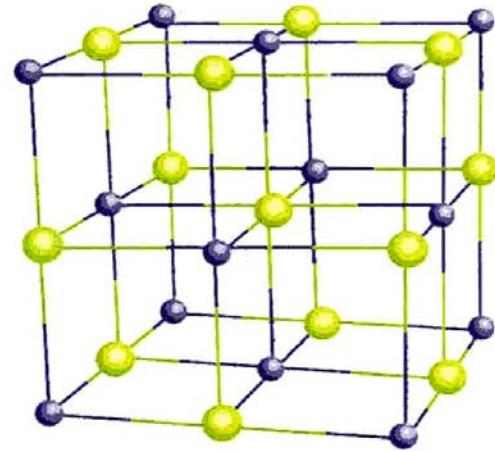
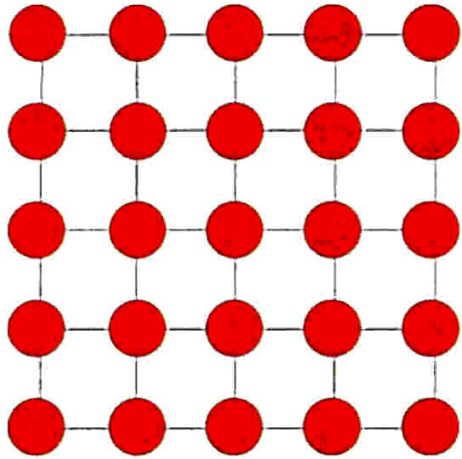
**Climate system:** nodes represent locations  
links similar climate



Brown-same operating system-now  
Wang et al (Science 2009)

Percolation-Immunization

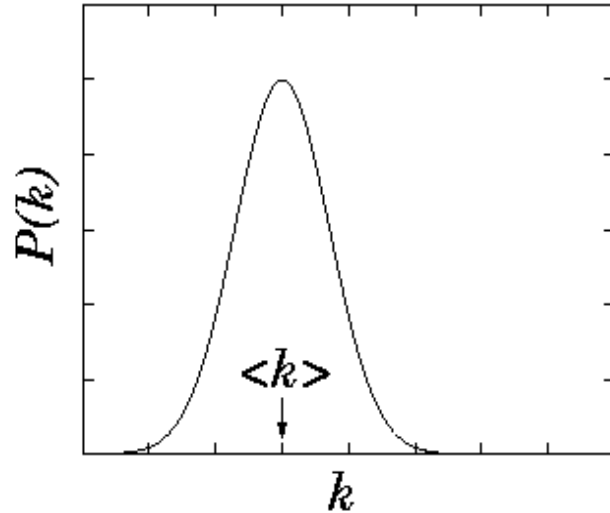
# Networks in Physics



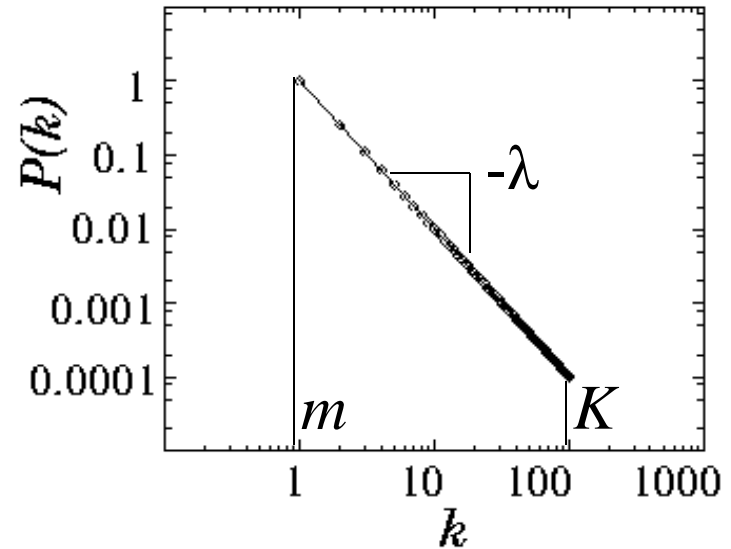


# Complex Single Networks- Since 2000

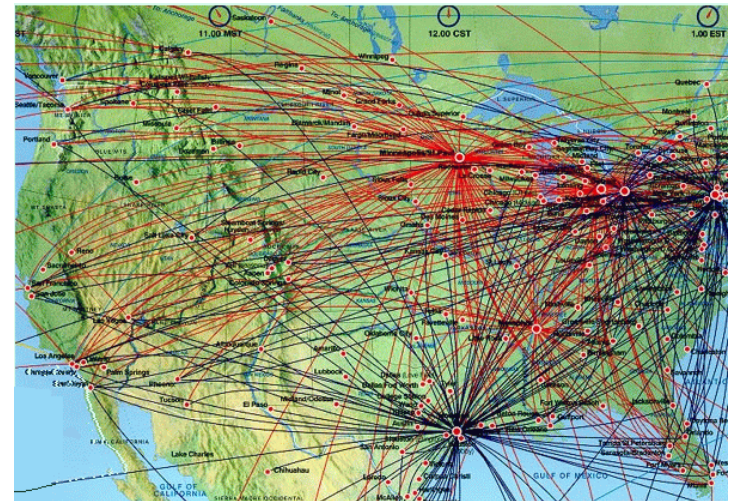
Poisson distribution



Scale-free distribution

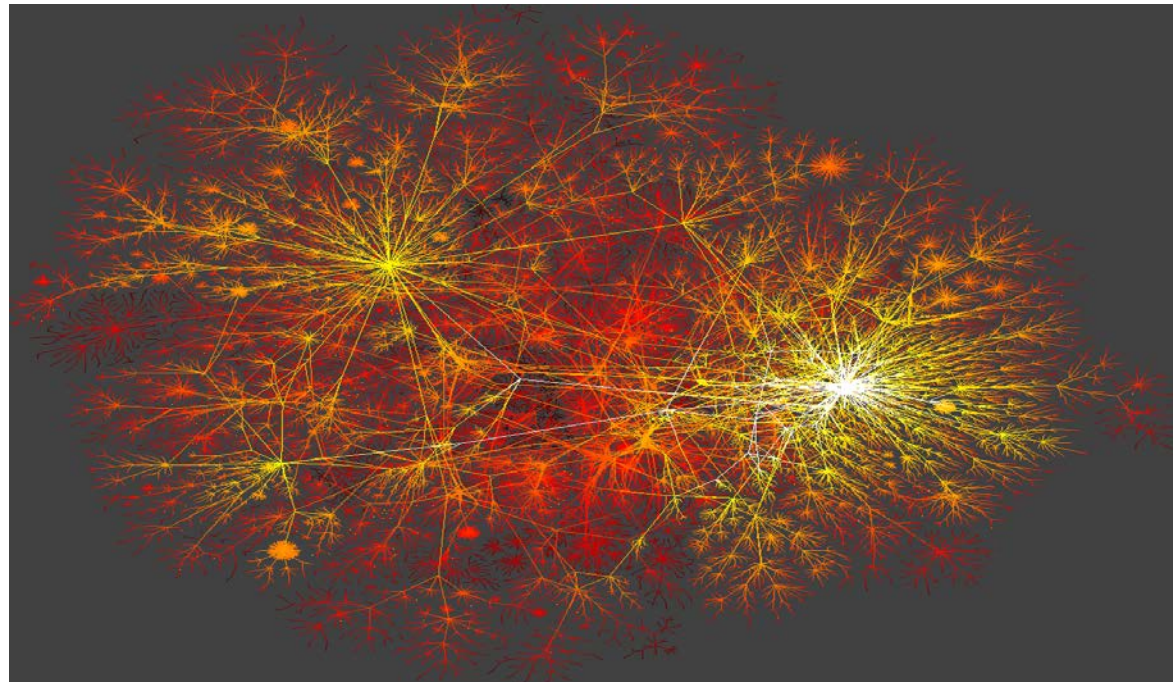
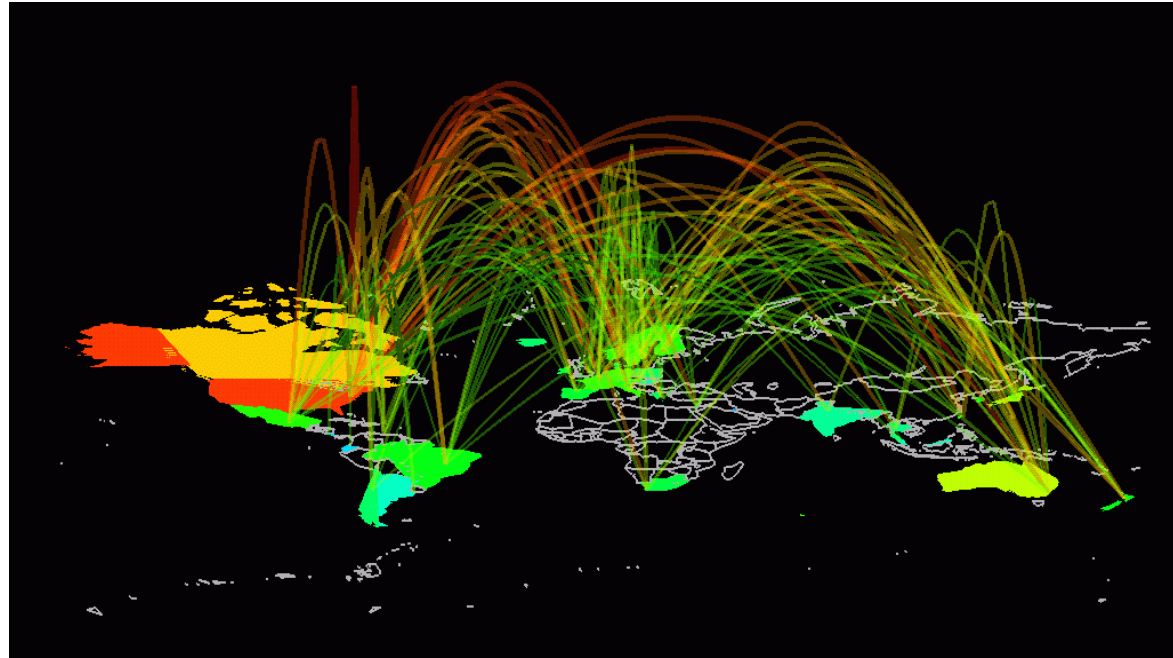
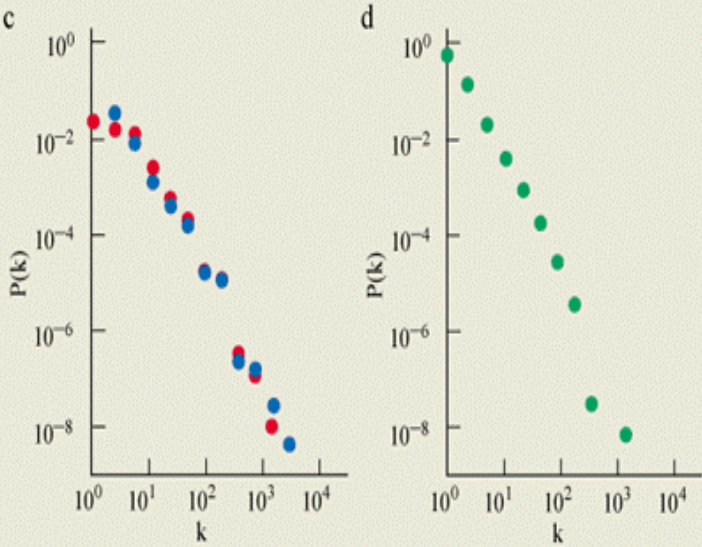
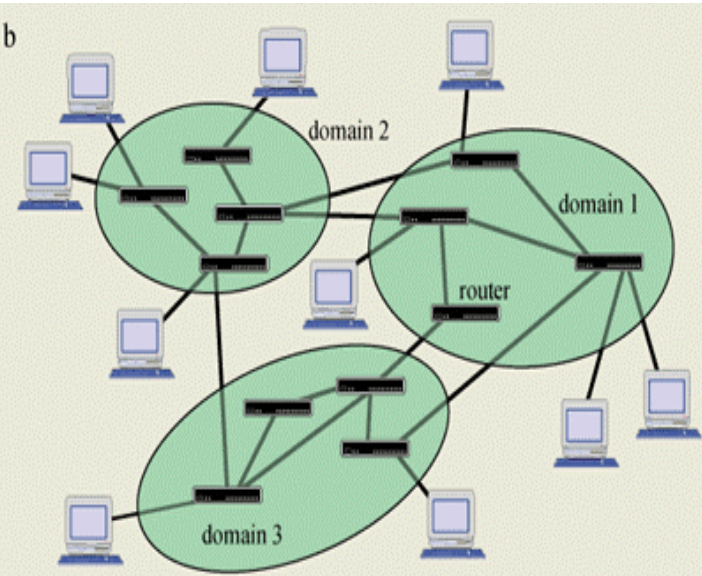


**Erdős-Rényi Network**



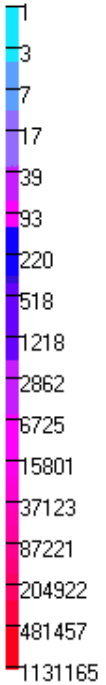
**Scale-free Network**

Faloutsos et. al., SIGCOMM '99



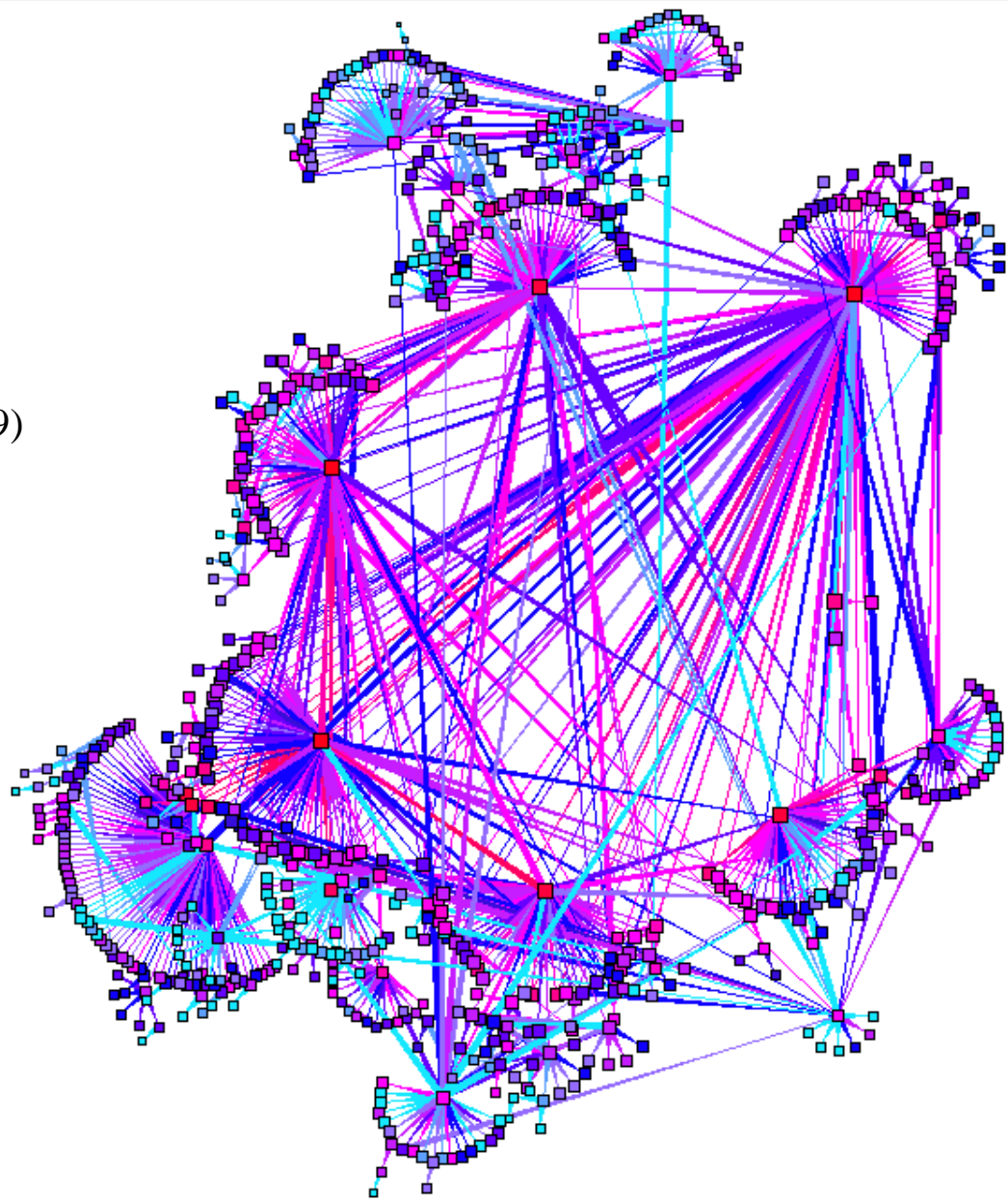


HTTP Requests



WWW-Network

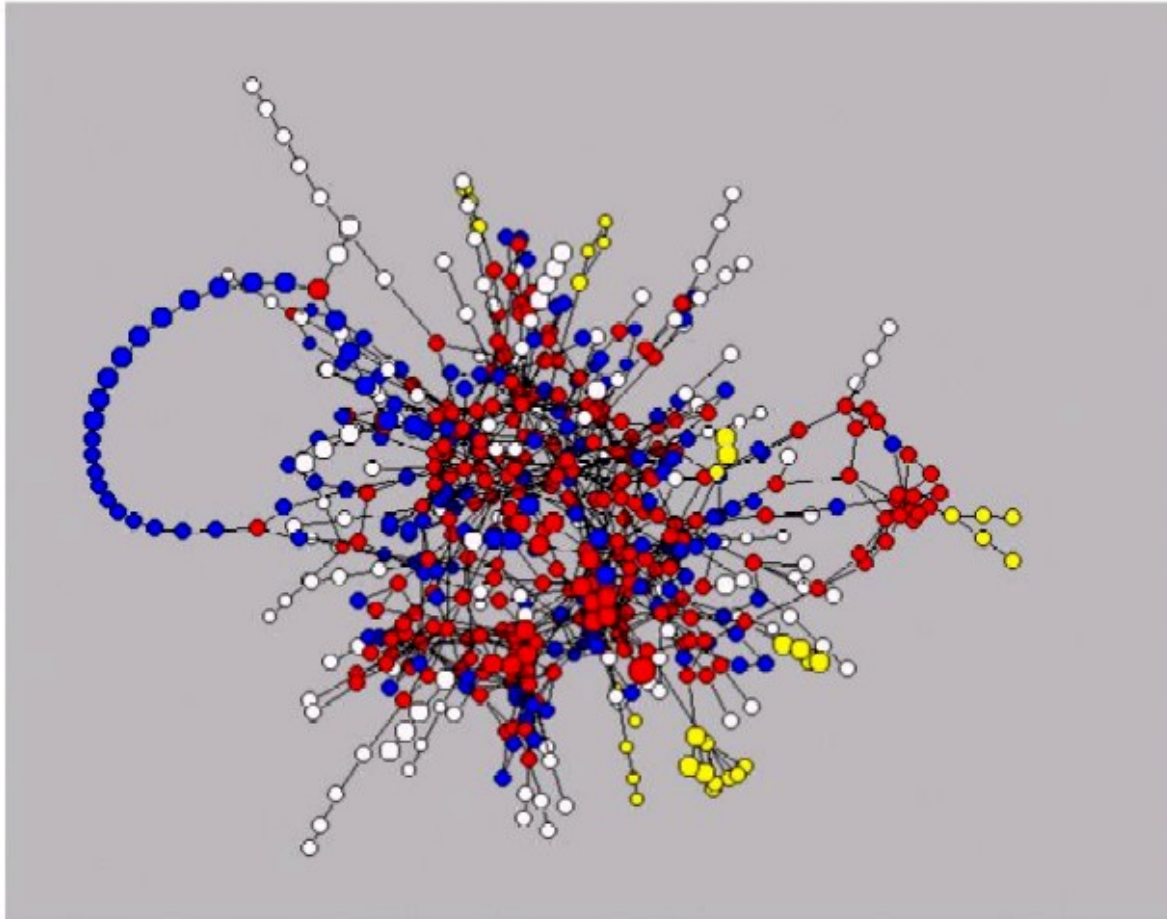
Barabasi et al (1999)



# Metabolic Network

Nodes: chemicals (substrates)

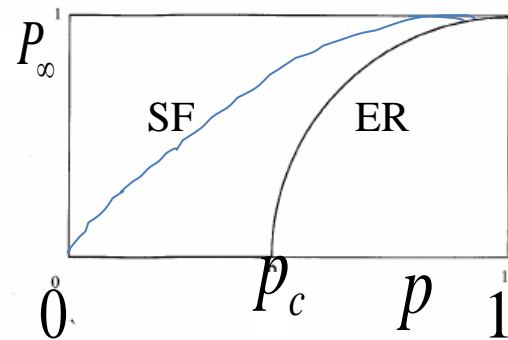
Links: bio-chemical reactions



Jeong, Tombor, Albert, Barabasi, Nature (2000)

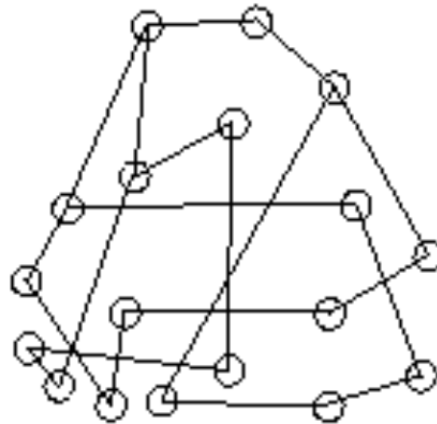


# Many real networks are non-Poissonian



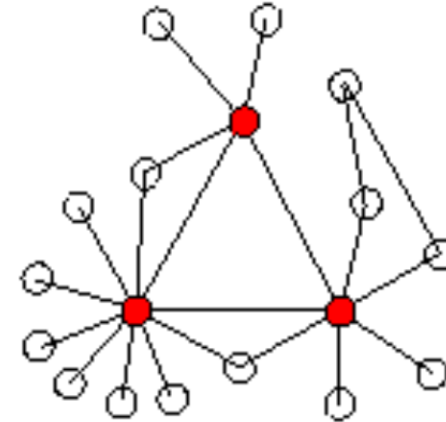
SF more robust!!

## Exponential



$$P(k) = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

## Scale-free



$$P(k) = \begin{cases} ck^{-\lambda} & m \leq k \leq K \\ 0 & \text{otherwise} \end{cases}$$

Classical Erdos-Renyi (1960)

Homogeneous, similar to lattices

$d \sim \log N$  -- Small world

$$p_c = 1 - q_c = 1 / \langle k \rangle$$

$$P_\infty = p[1 - \exp(-\langle k \rangle P_\infty)]$$

Barabasi-Albert (1999)

Heterogeneous-translational symmetry breaks!

Change universality class-many anomalous laws

$$e.g., \quad d \sim \log \log N ; p_c = 0$$

Ultra Small worlds (Cohen and SH, PRL (2003))

Breakthrough in understanding many problems!

# WHAT IS DIFFERENT?

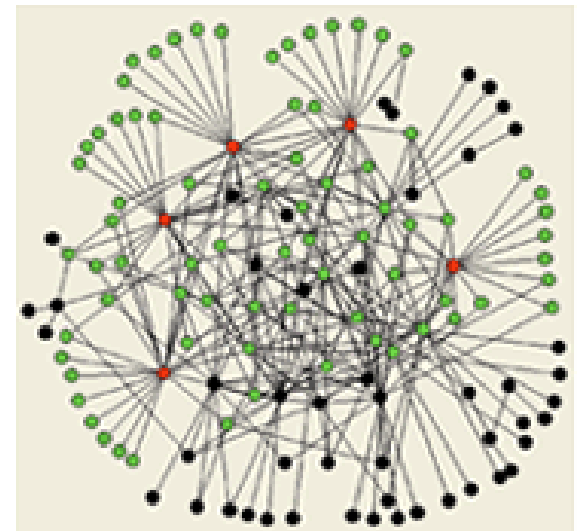
## Known values of immunization thresholds:

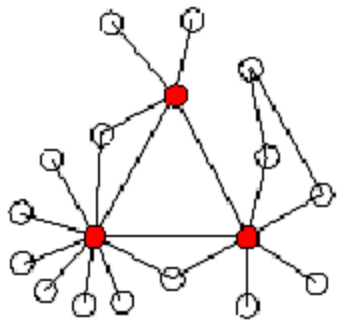
Infectious disease	Critical Threshold $q_c = 1 - p_c$
Malaria	99%
Measles	90-95%
Whooping cough	90-95%
Fifths disease	90-95%
Chicken pox	85-90%
Internet	more than 99%

Such immunization thresholds were **not understood** since they were well above the expected value of percolation in classical random networks:

$$q_c = 1 - p_c = 1 - 1 / \langle k \rangle$$

This puzzle is solved due to the broad degree distribution (HUBS) of social networks which does not occur in random graphs!

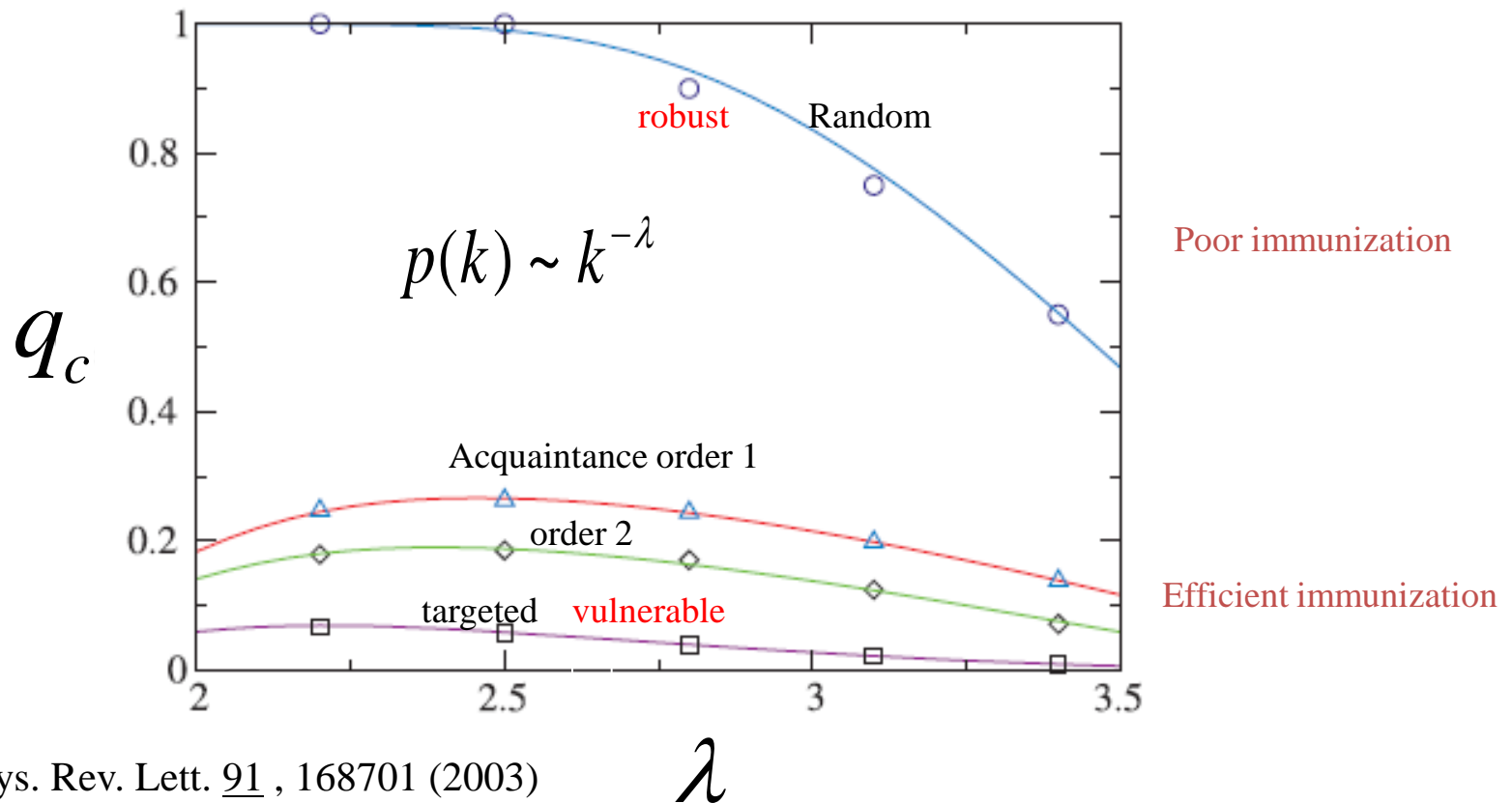




## Scale Free networks --immunization strategies

### Efficient Immunization Strategy: Acquaintance Immunization

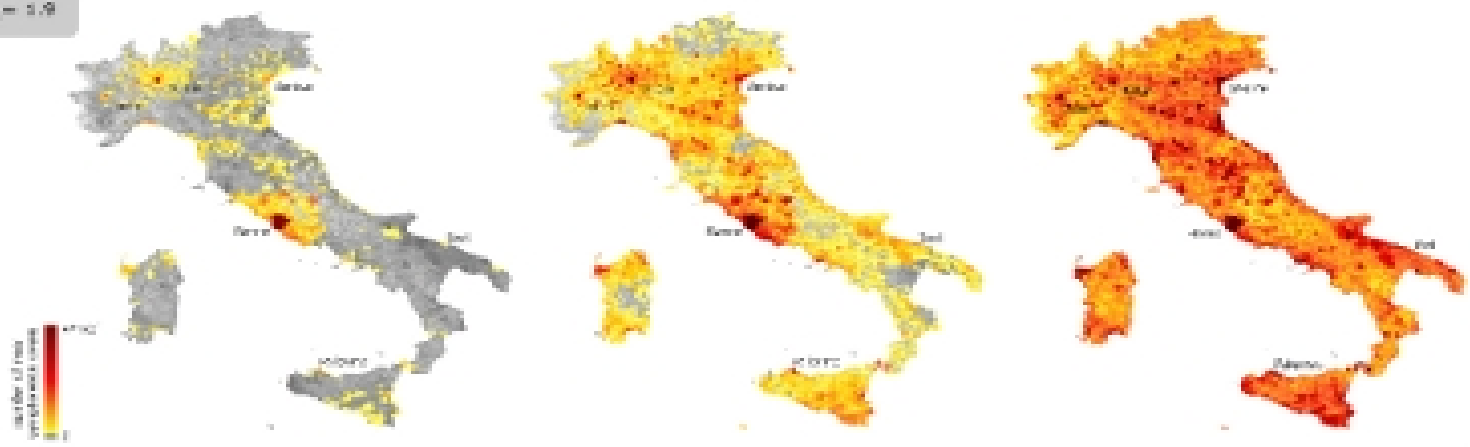
$q_c$  – critical fraction of removed or immunized nodes





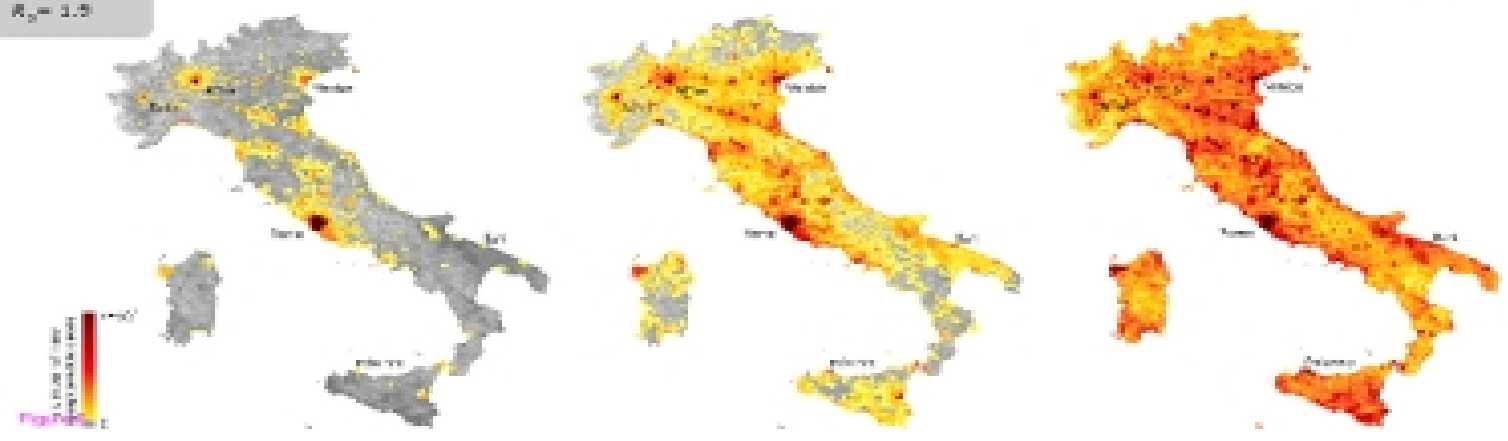
# EPIW<sup>IR</sup>K

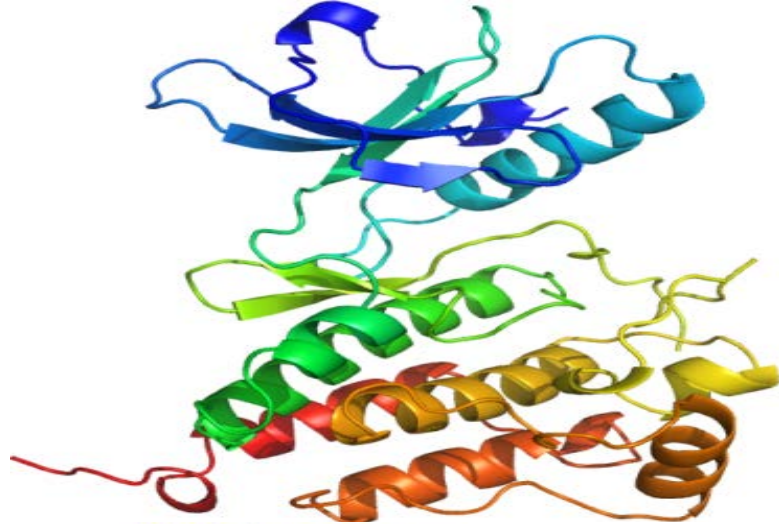
GLEaM  
 $R_0 = 1.9$



127 days      148 days      178 days      time: number of days since the first large-batch of cases

Agent-based  
 $R_0 = 1.9$





### :Protein sequence

HASEEENDWQEELEAQEDNAPAEELSDSDAPLKPND  
 TRKGEKGRKKKKRKNSEEDSDFVCHDEEVEYPSKSR  
 MPSVEDVCSAPSVCHVEIEYSEELQSLTTYKAFSHVVRP  
 KWREPCESNPHIQQEGGAAGSGSAGQARSVTGDEPEPR  
 EEEEEEEKKPRKRSGRQKGRRPQKVPVLKIKLLGKR  
 LEFERMLQKSDSDADEKEAPVSSKADNSAPAAQDDGSGAP  
 KKTGNPPIGEDGHHHQDYCEVCQGGGEIILCDTCPRAYH  
 CRADGGAAEEEDDDEHQBFRCVCKDGGELLCCD

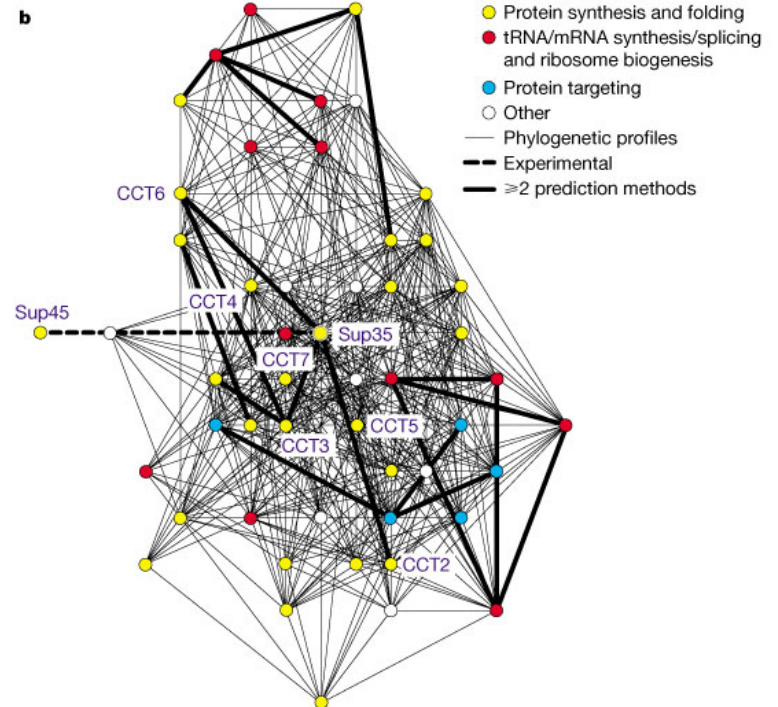
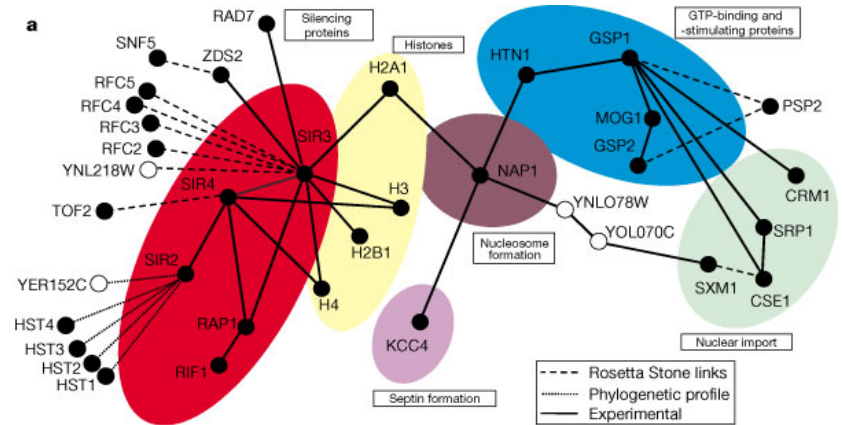
# Function Prediction??



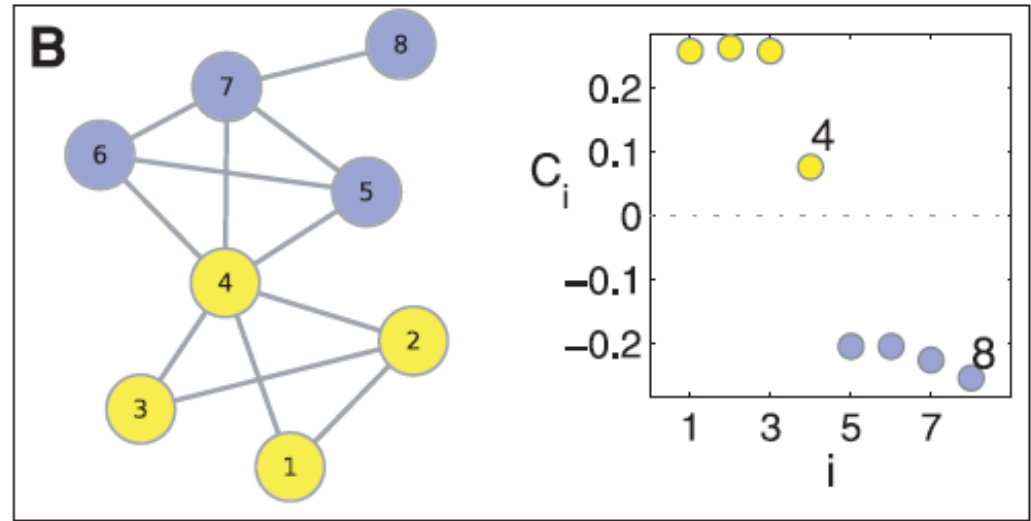
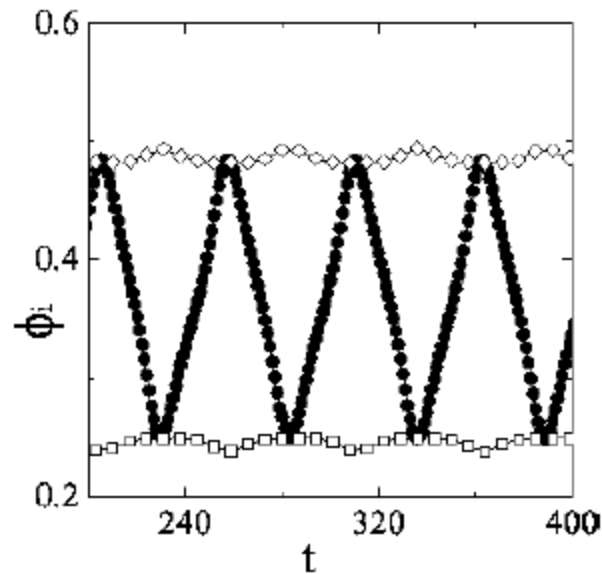
:Protein structure



:PPI-Network



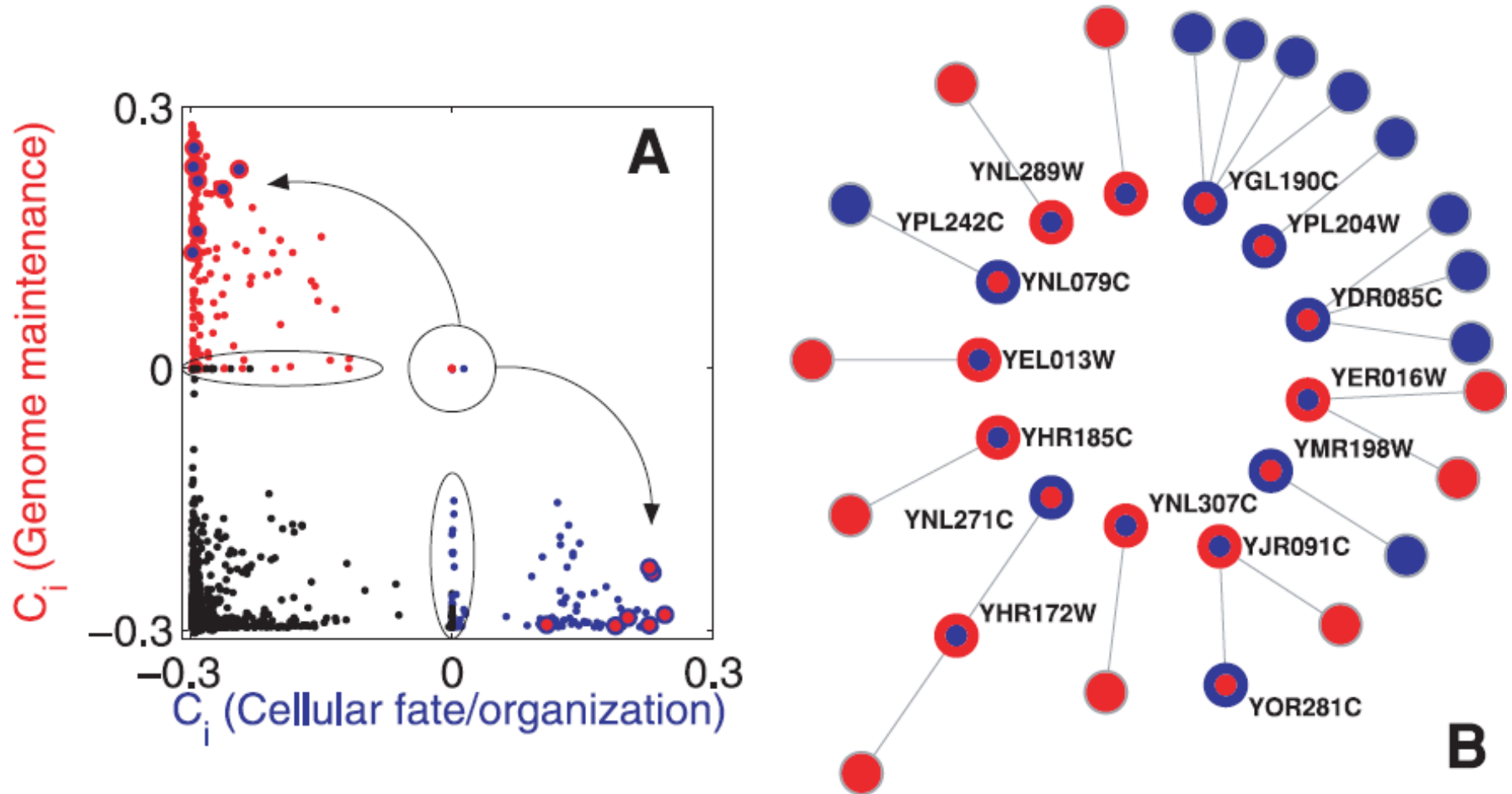
# Unveiling Protein Functions by Synchronization in the Interaction Network



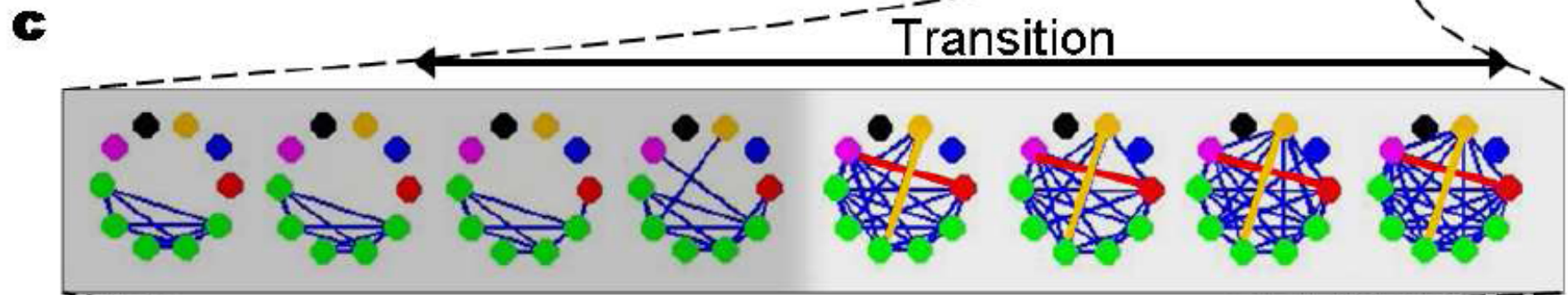
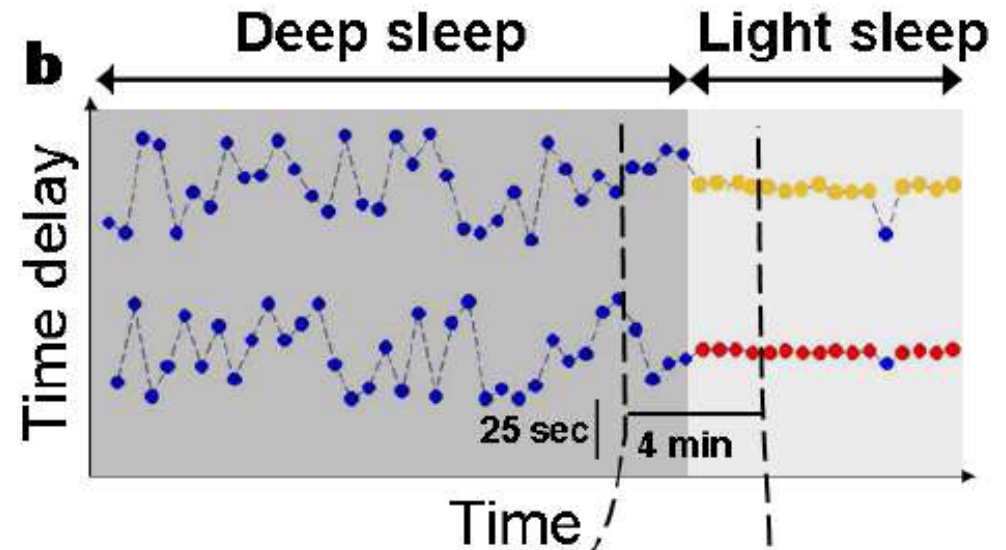
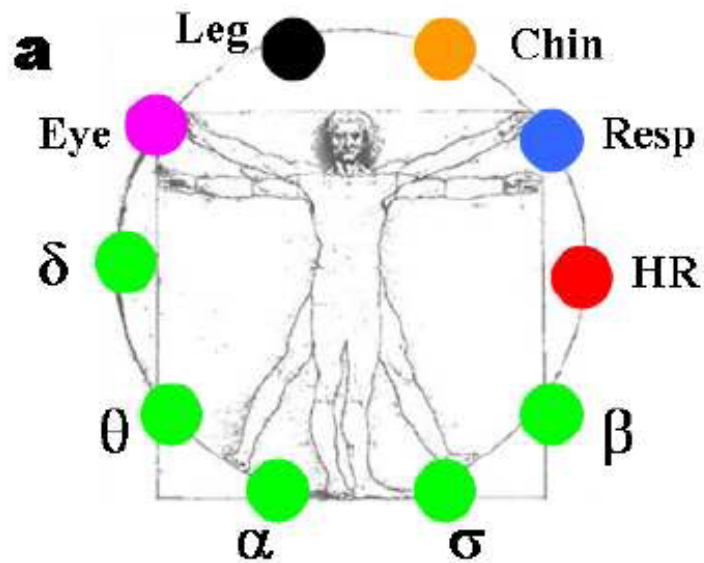
Irene Sendin~a-Nadal, Yanay Ofra, Juan A. Almendral<sup>1</sup>, Javier M. Buldu, Inmaculada Leyva, Daqing Li, Shlomo Havlin, Stefano Boccaletti, Plos One (2011)



# Unveiling Protein Functions by Synchronization in the Interaction Network



Irene Sendin~a-Nadal, Yanay Ofran, Juan A. Almendral, Javier M. Buldu, Inmaculada Leyva, Daqing Li, Shlomo Havlin, Stefano Boccaletti, Plos One (2011)



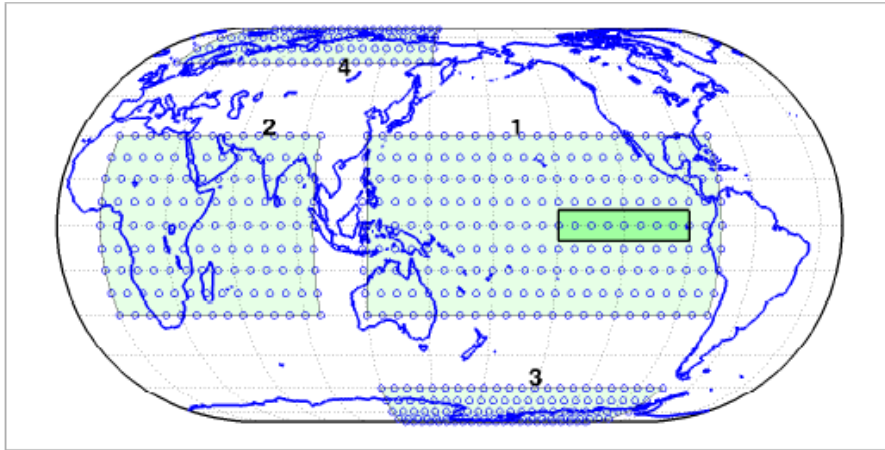
Bashan et al, Nature Communication [2012]

Structure and Function

Makse et al, PNAS (2012) optimized transport in brain

(Andrade et al PRL 2010, PRE (2013))

# Climate networks are very sensitive to El Nino

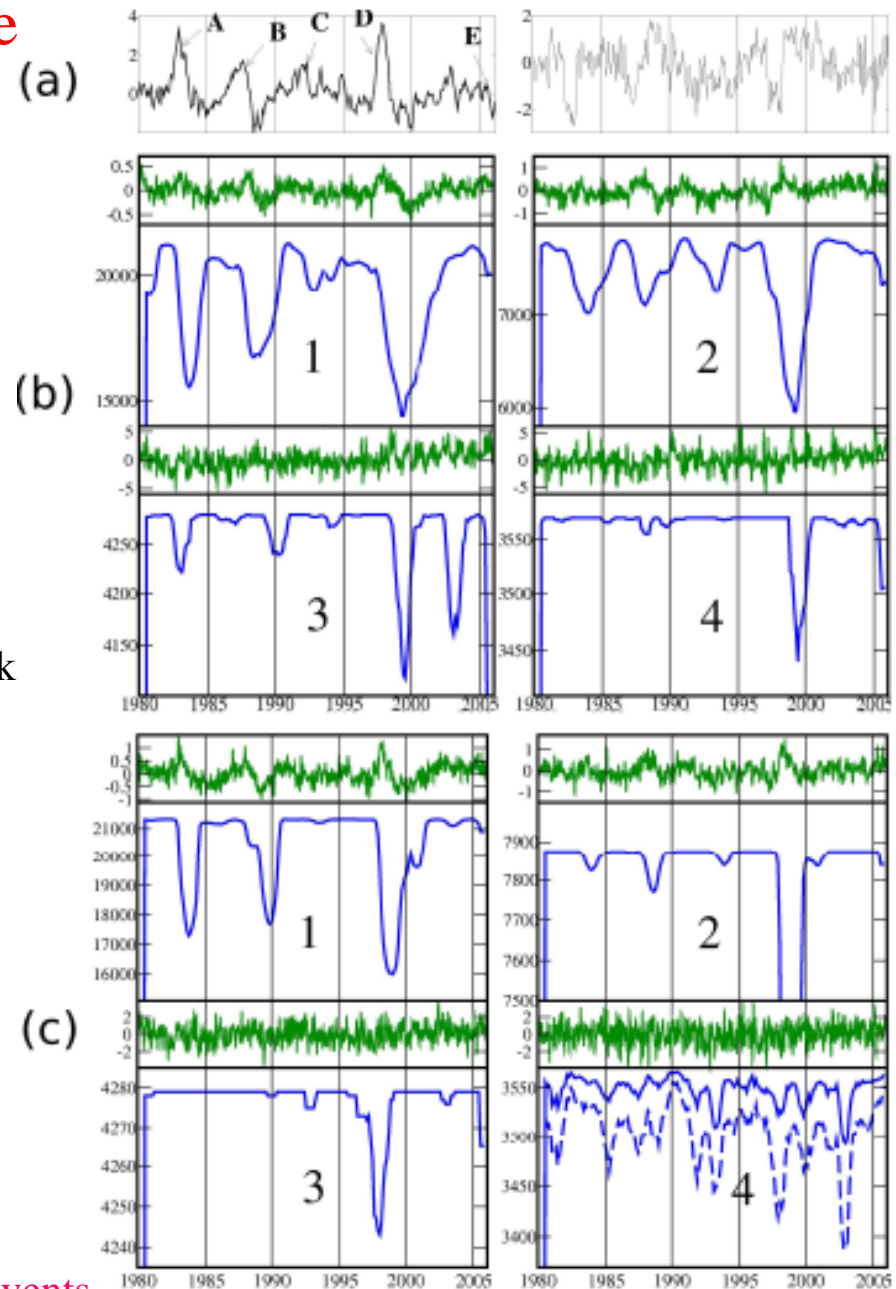


Sea surface temperature network

5km height temperature network

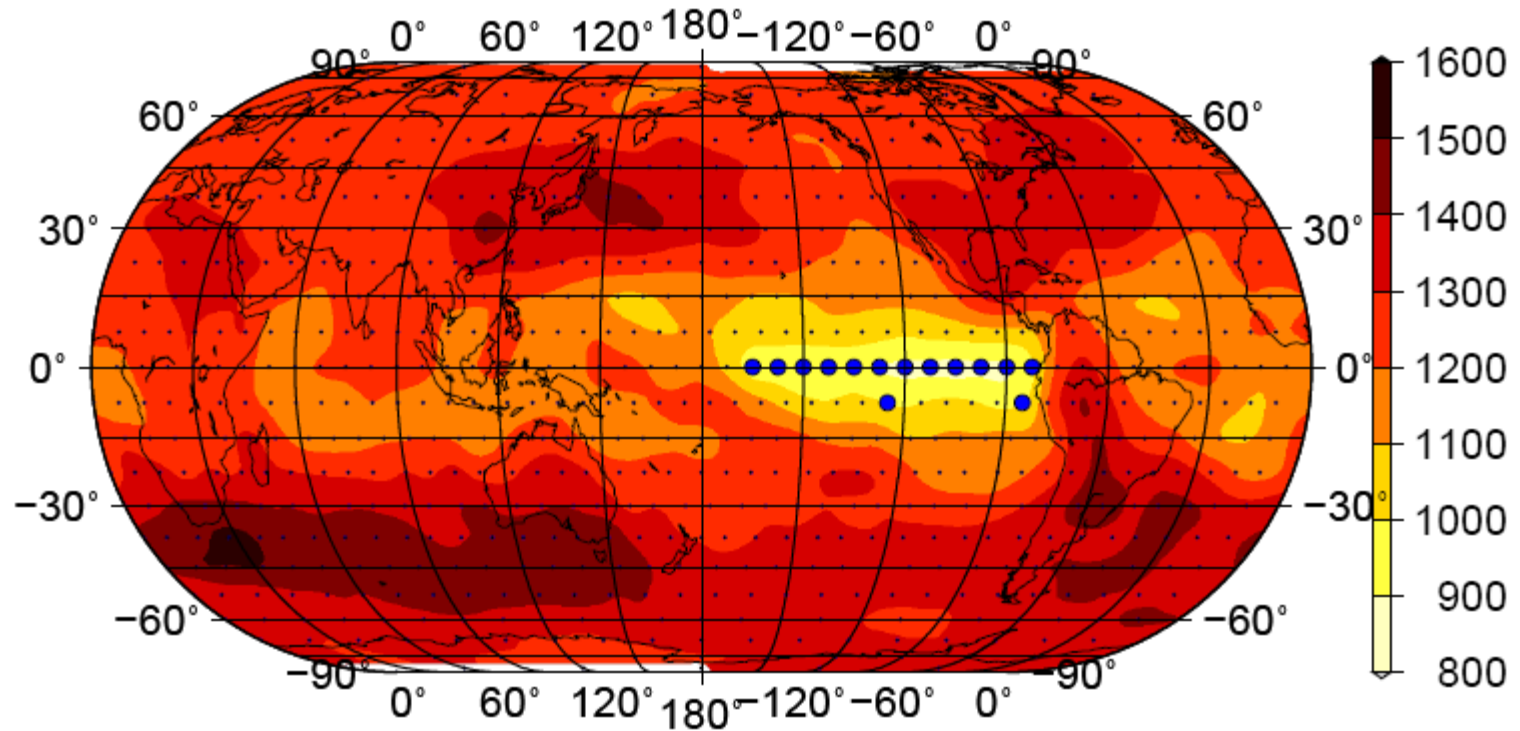
Yamasaki, Gozolchiani, SH (PRL 2008, 2011)

Challenge: Predicting El-Nino and other extreme events





# EL-NINO BECOMES AUTONOMOUS: ONLY INFLUENCE-NOT INFLUENCED



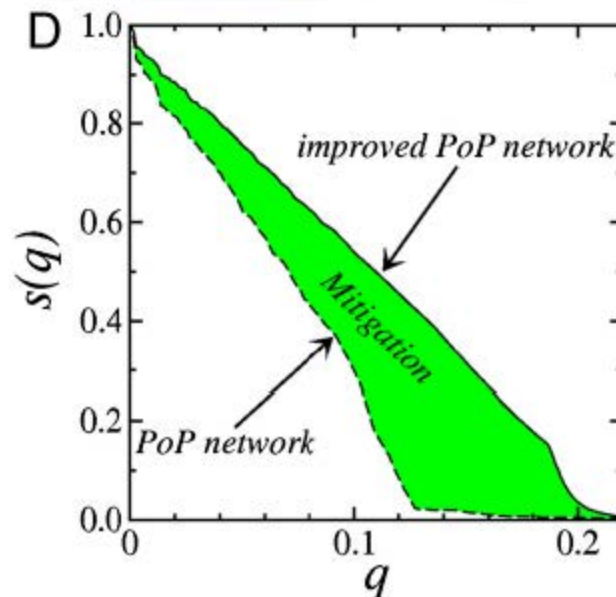
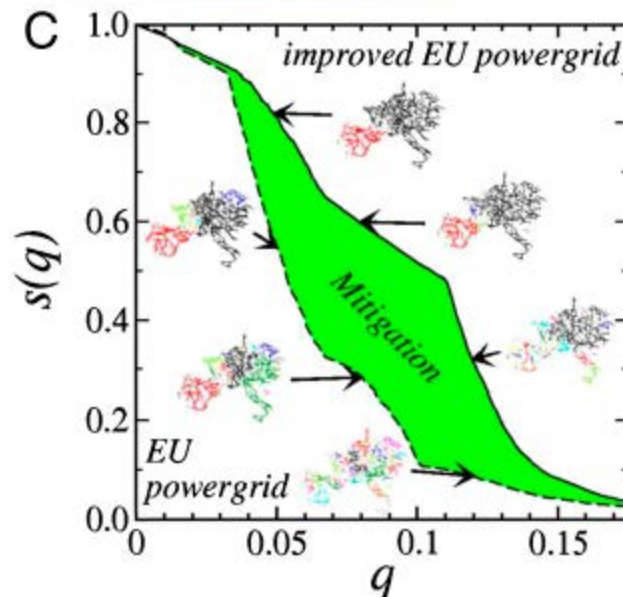
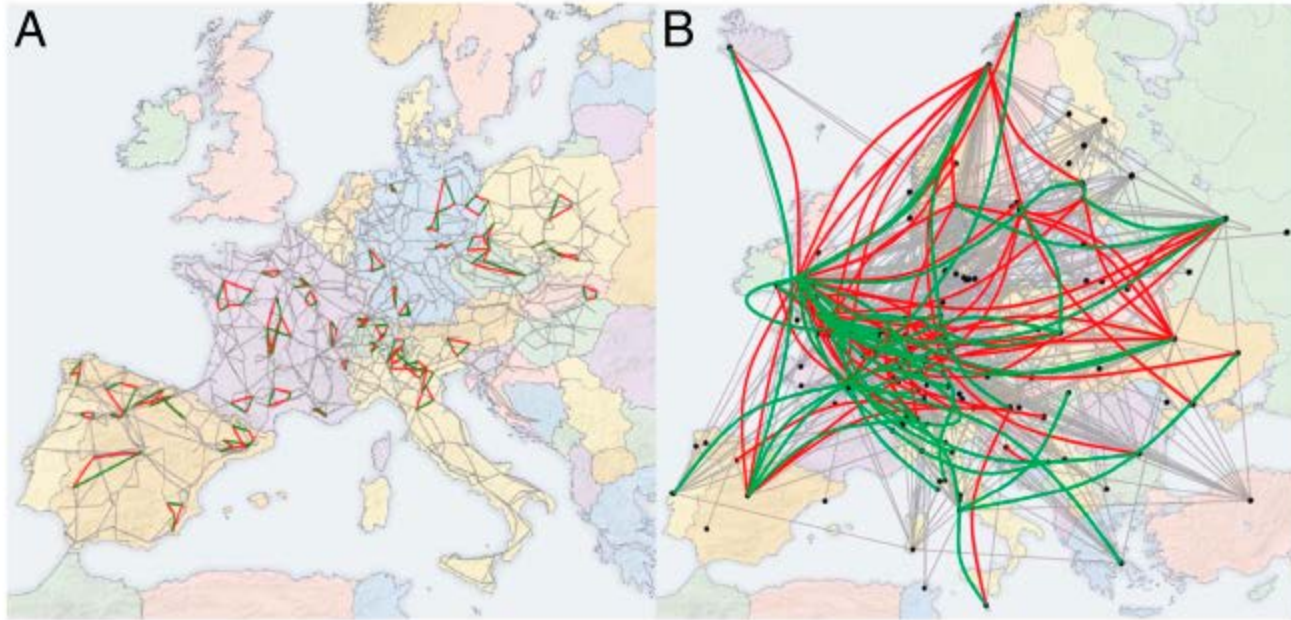
$$\langle I_l^y \rangle_{y \in El-Nino}$$

Gozolchiani et al PRL (2011)

# Mitigation of malicious attacks on networks

Power Grid

Internet



Schneider, Moreira,  
Andrade, SH  
and Herrmann  
PNAS (2011)



VOLUME 279  
NUMBER 73

Suggested retail price  
**\$1.00**  
\$1.50 outside of  
Metro Boston

# The Boston Globe

MONDAY, MARCH 14, 2011

A NEW WEEK

TODAY: Partly sunny and colder. High 37-42. Low 27-32.

TOMORROW: Mostly sunny, mild. High 42-47. Low 32-37.

HIGH TIDE: 6:42 a.m., 7:25 p.m.  
SUNRISE: 6:59 SUNSET: 6:49

FULL REPORT: PAGE B13

## Cascading disaster in Japan

### Blast shakes a second reactor death toll soars

By Martin Fackler  
and Mark McDonald  
NEW YORK TIMES

SENDAI, Japan — Japan reeled from a rapidly unfolding disaster epic scale yesterday, pummeled by a death toll, destruction, and homelessness caused by the earthquake and tsunami and new hazards from damaged nuclear reactors. The prime minister called it Japan's worst crisis since World War II.

Japan's \$5 trillion economy, the world's third largest, was threatened with severe disruptions and partial paralysis as many industries shut down temporarily. The armed forces and volunteers mobilized for the far more urgent crisis of finding survivors, evacuating residents near the stricken power plants and caring for the victims of the record 8.9 magnitude quake that struck on Friday.

The disaster has left more than 10,000 dead, many thousands homeless, and millions without water, power, heat, or transportation.





# Catastrophic Cascading Failures in Interdependent Networks

## Work with:

A. Bashan (BIU)

Y. Berezin (BIU)

S. Buldyrev (NY)

R. Parshani (BIU)

J. Gao (BU)

H. E. Stanley (BU)

**Nature, 464, 1025 (2010)**

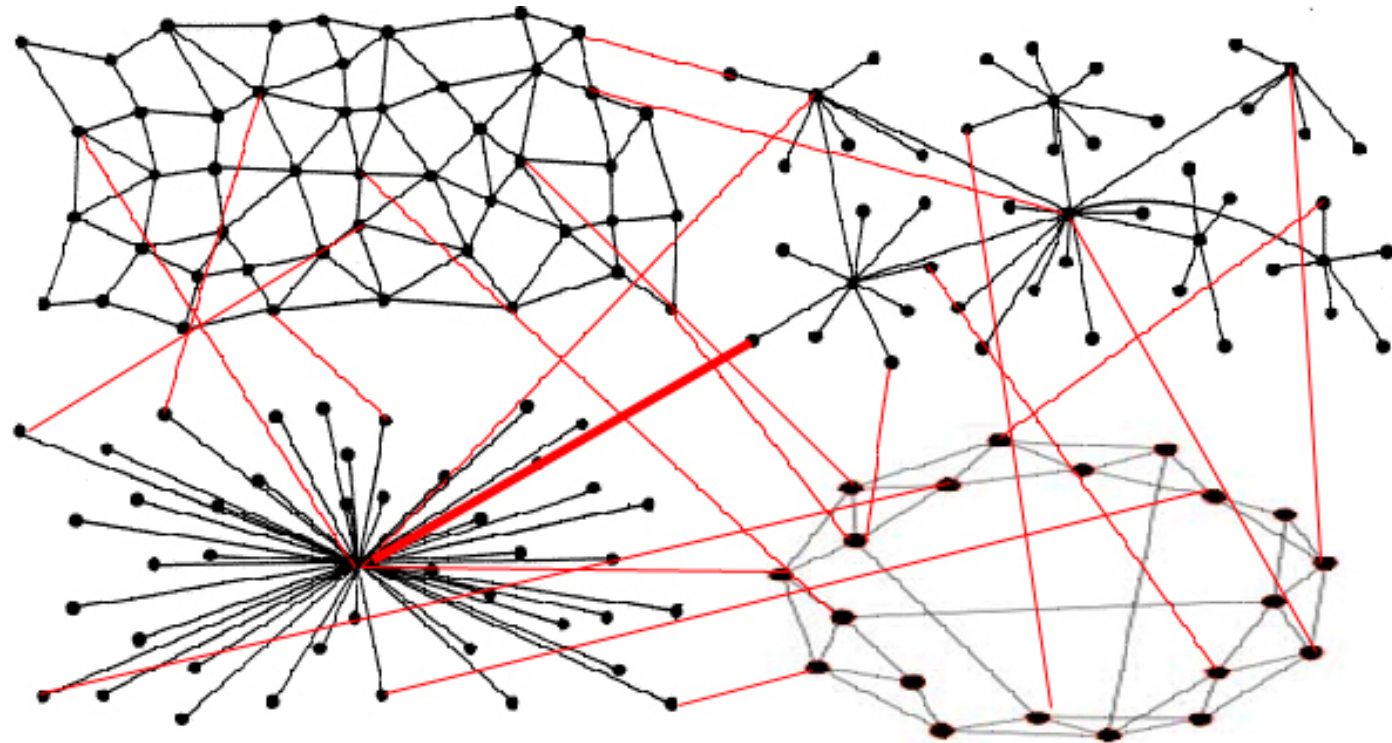
**PRL, 105, 0484 (2010)**

**PNAS, 108, 1007 (2011)**

**PRL, 107, 195701 (2011)**

**Nature Physics (2012)**

**Nature Comm. (2012)**



Electric grid,  
Communication  
Transportation  
Services .....

Two types of **links**:

1. Connectivity

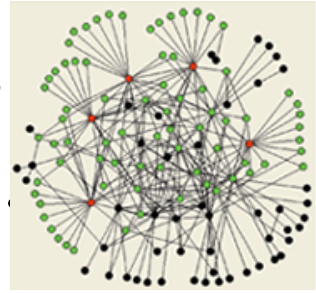
2. **Dependency**

**Cascading disaster**

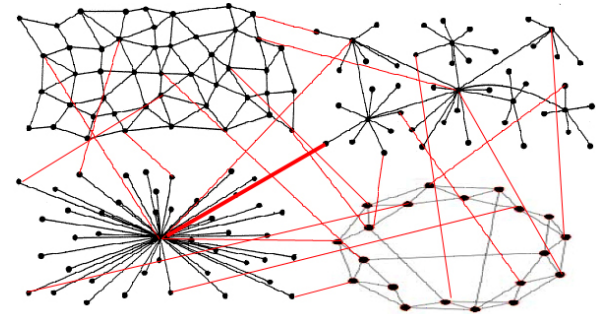
Raissa D'sousa-same type

# Interdependent Networks

- Until **recently (2010)** studies focused on a **single network** which is isolated AND does not interact or influenced by other systems.

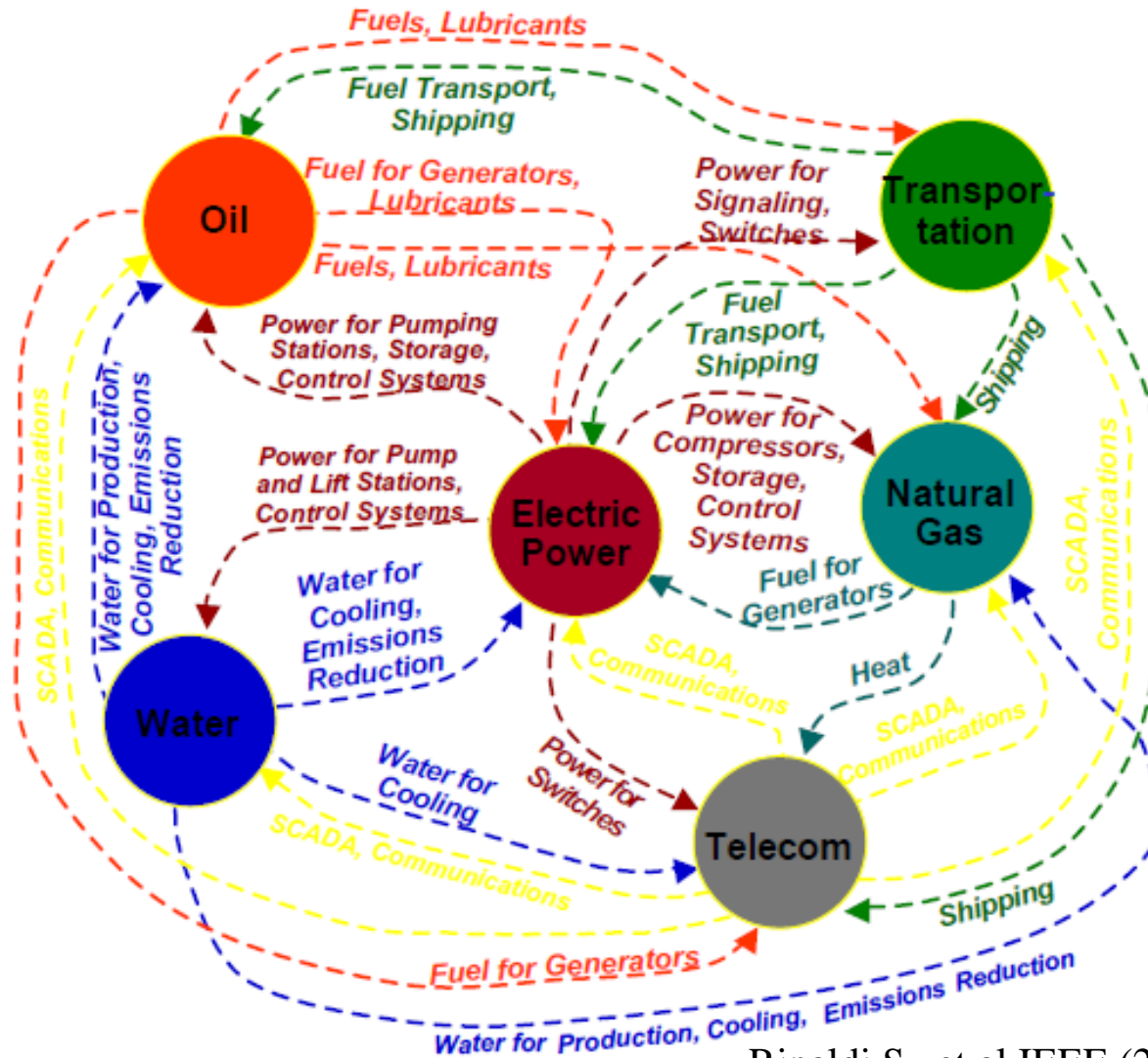


- Isolated systems **rarely** occur in nature or in technology -- analogous to **non-interacting** particles (molecules, spins).



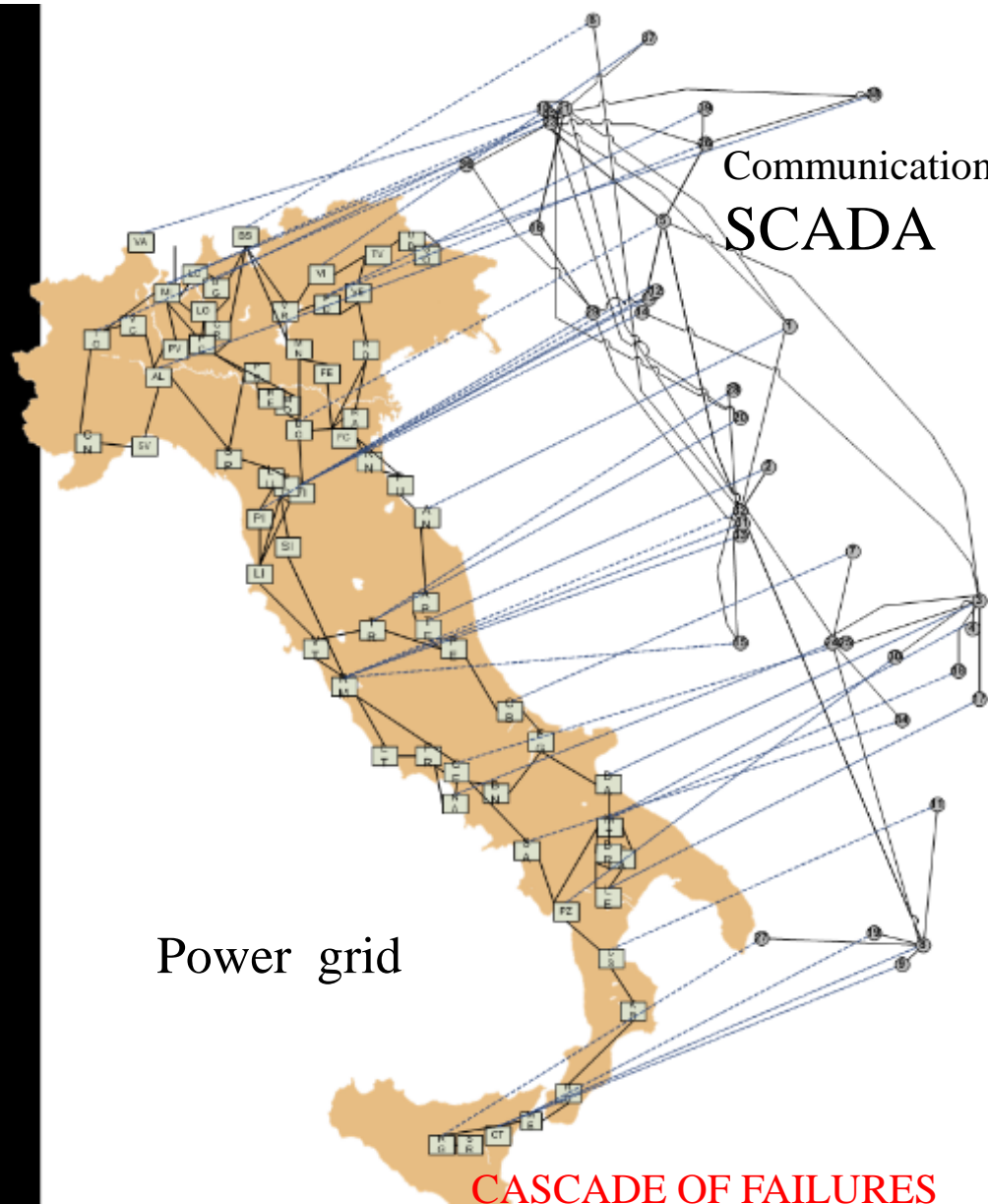
- Results for **interacting networks** are strikingly **different** from those of single networks.

## How interdependent are infrastructures?



Rinaldi S, et al IEEE (2001)

# Blackout in Italy (28 September 2003)



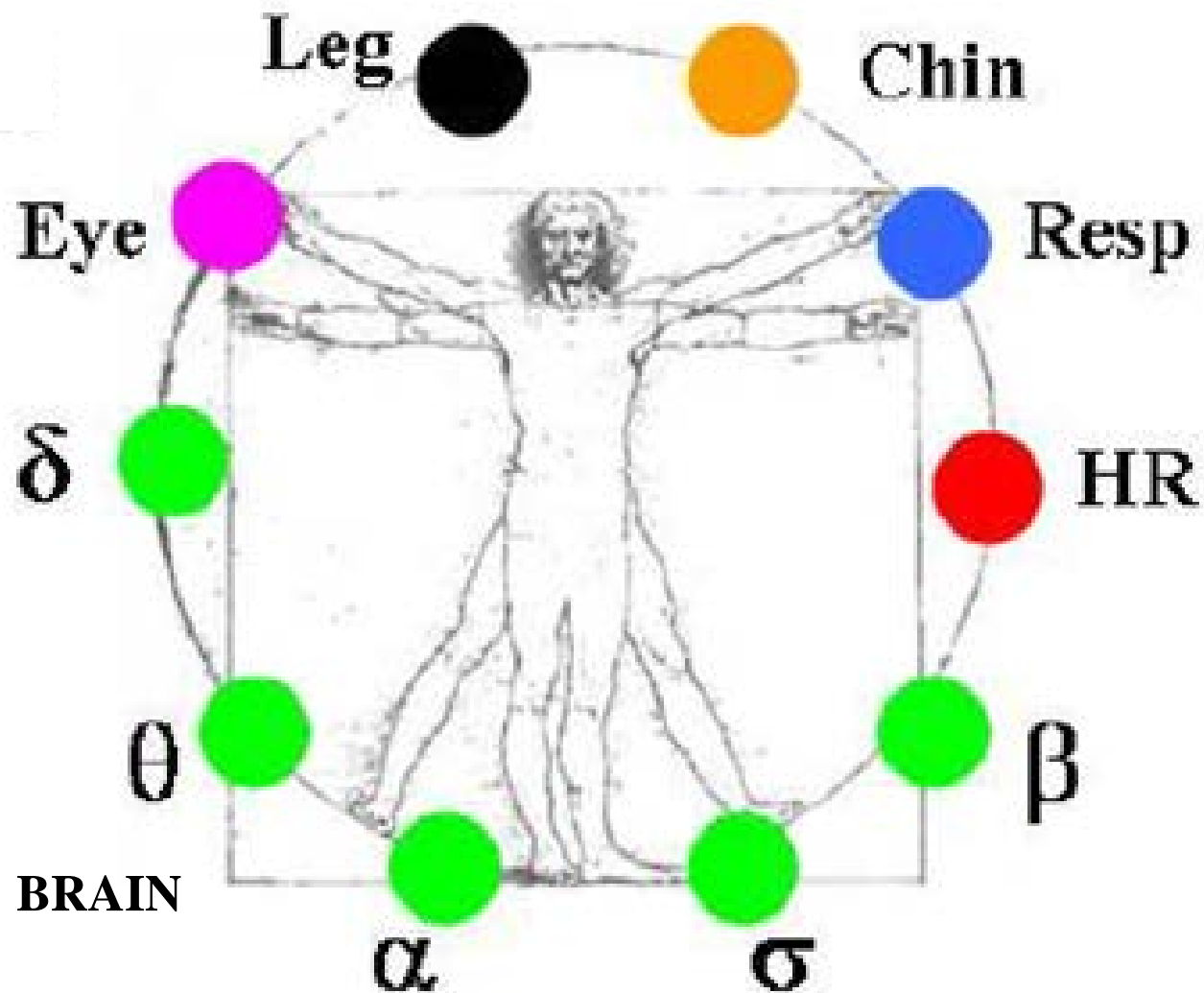
Cyber  
Attacks-  
CNN  
Simulation  
(2010)

Rosato et al  
Int. J. of Crit.  
Infrastruct. 4,  
63 (2008)

Railway network, health care systems, financial services, communication systems

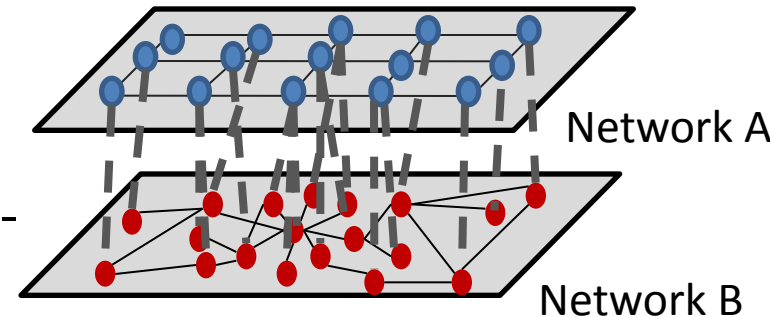


# HUMAN BODY: NETWORK OF NETWORKS



# Further Examples of Interdependent Networks

Appear in all aspects of life, nature and technology



- *Physiology*: The human body can be seen as inter-dependent networks. For example, the cardiovascular network system, the respiratory system, the **brain** network, and the nervous system all depend on each other.
- *Biology*: A specific cellular function is performed by a network of interacting proteins. Such networks depend on each other through proteins that perform several functions.
- *Transportation* : Railway networks, airline networks and other transportation systems are interdependent.

## Critical Breakdown Threshold of Interdependent Networks

Failure in network A

causes failure in B, C, D... → causes further failure in A .....**CASCADES**

What are the critical percolation thresholds for such interdependent networks?

What are the sizes of cascade failures?

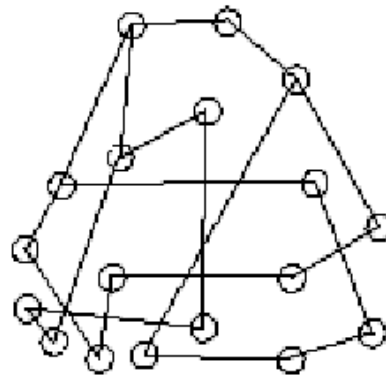
# Comparing single and coupled networks: Robustness

Remove randomly (or targeted) a fraction  $1 - p$  nodes

$P_\infty$  Size of the largest connected component (cluster)

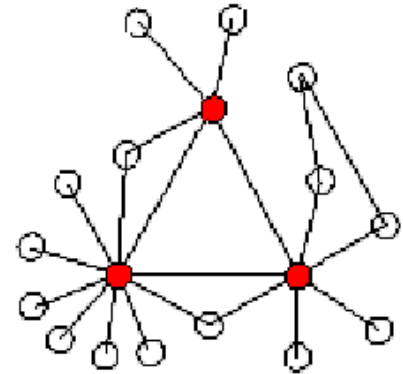
$p_c$  Breakdown threshold

**Exponential (ER)**



$$P(k) = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

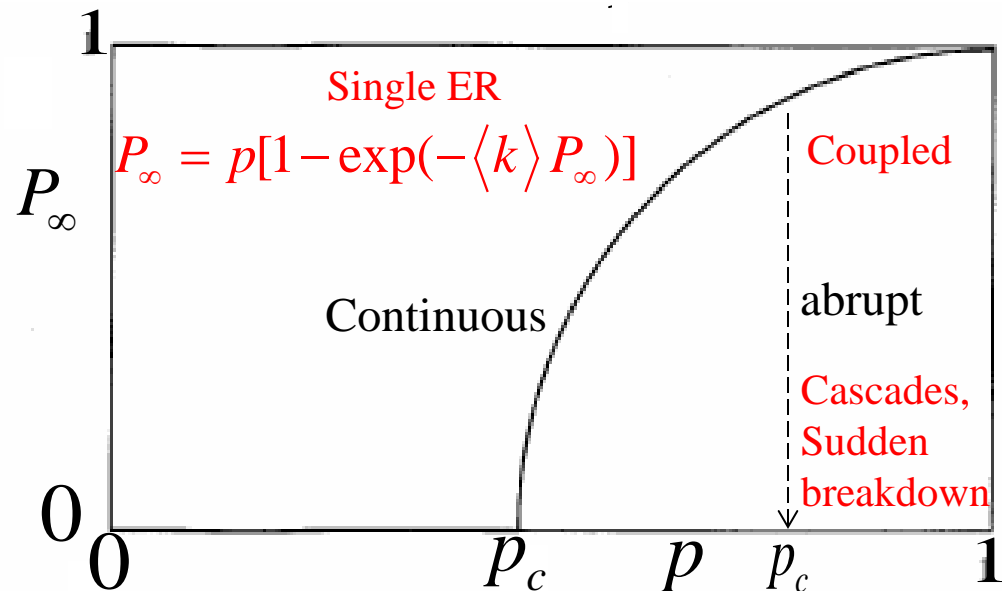
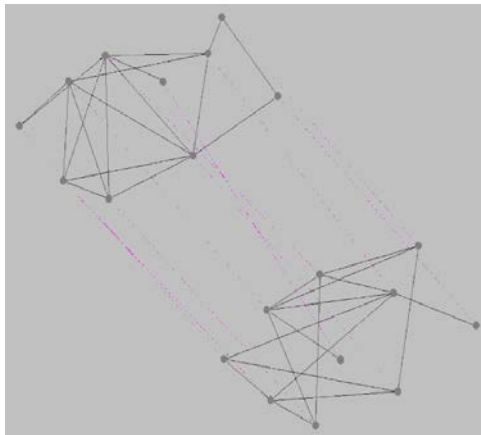
**Scale-free (SF)**



$$P(k) = \begin{cases} ck^{-\lambda} & m \leq k \leq K \\ 0 & \text{otherwise} \end{cases}$$

**Single networks:**

**Continuous transition**

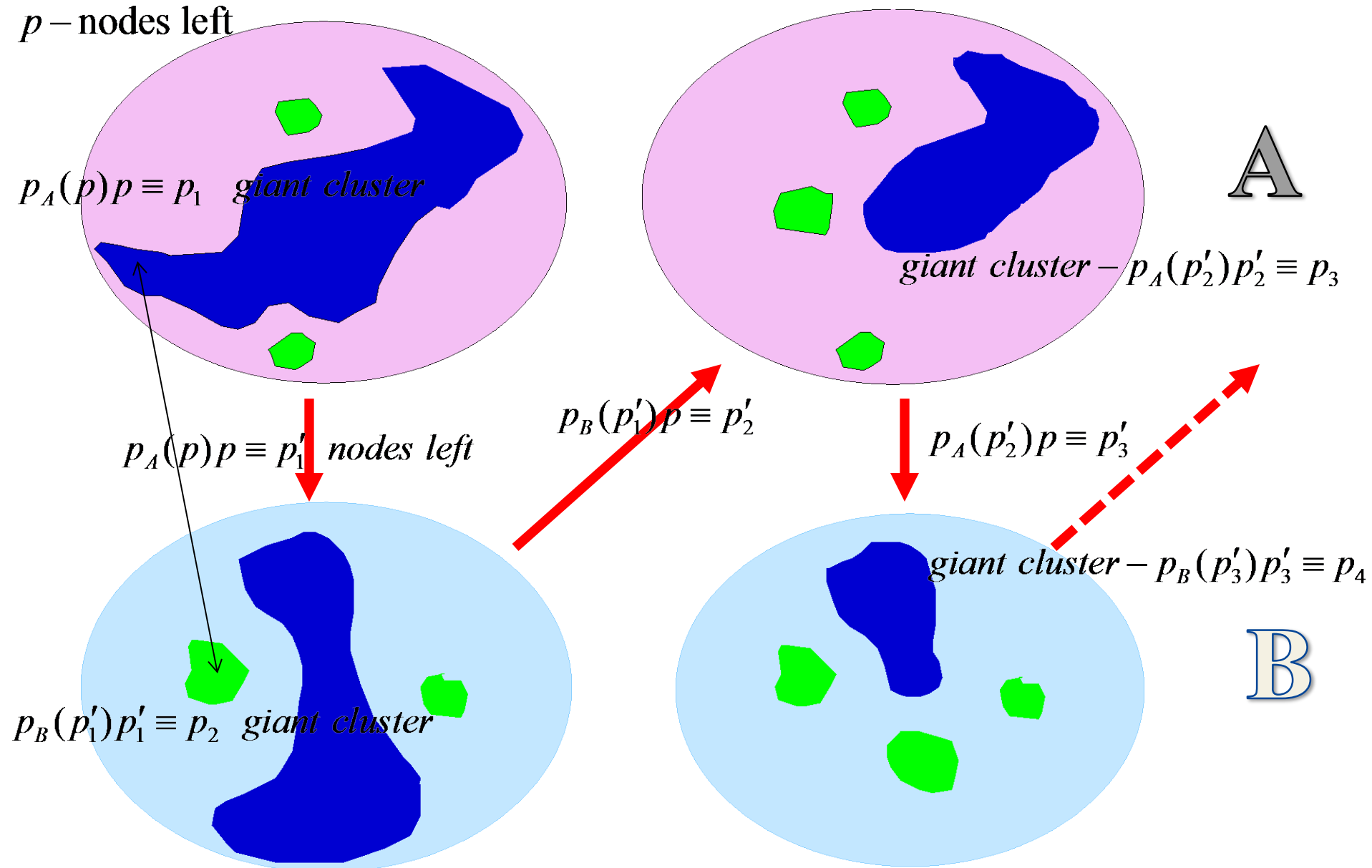


**Coupled networks:**

**New paradigm-Abrupt transition**

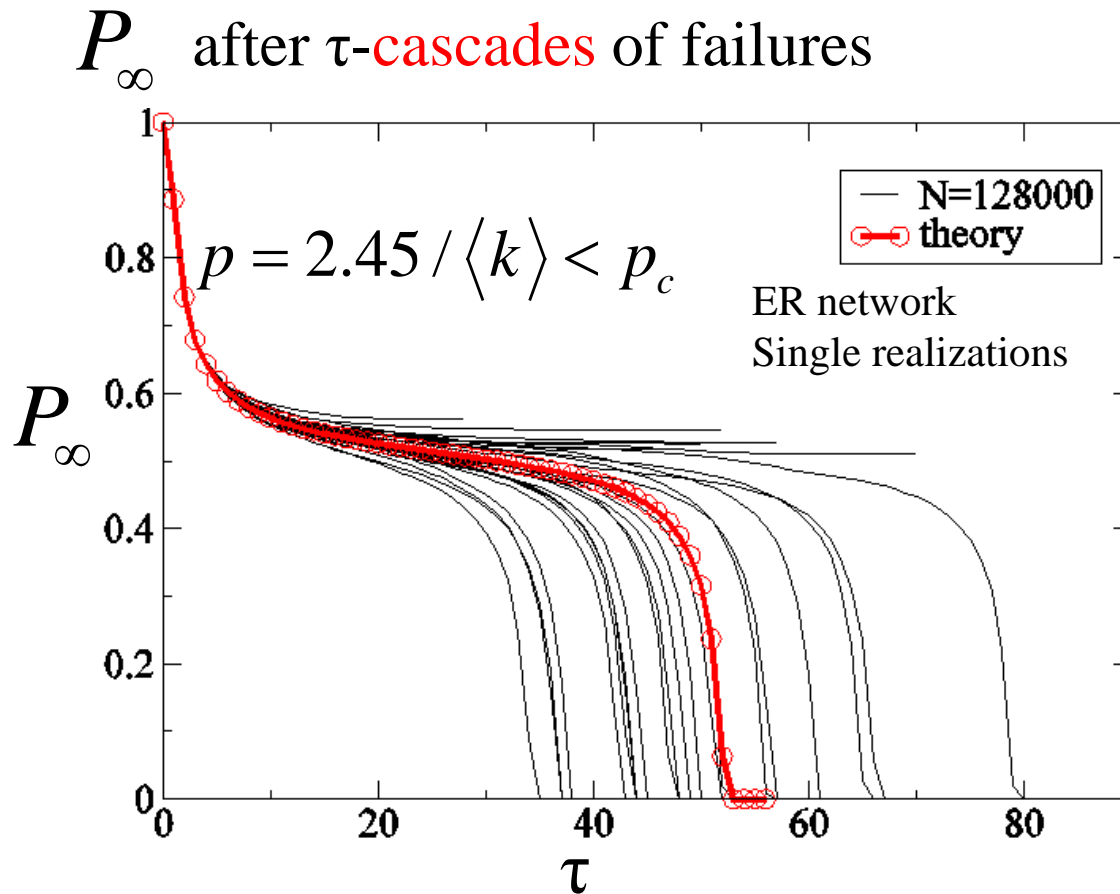
**Cascading Failures**

# RANDOM REMOVAL – PERCOLATION FRAMEWORK

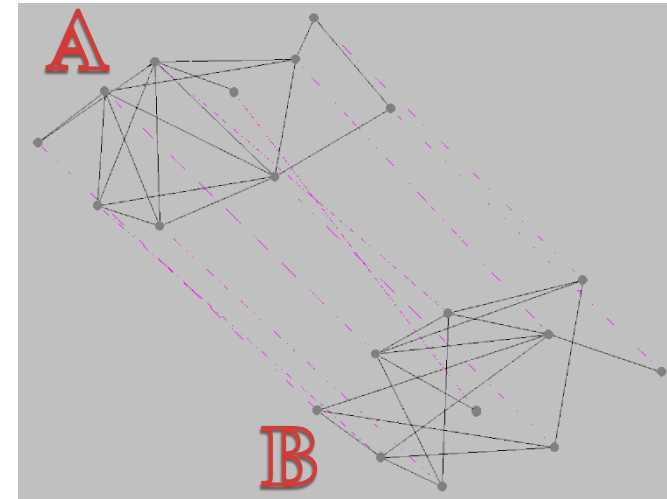




# RESULTS: THEORY and SIMULATIONS: ER Networks



Removing 1-p nodes in A



Catastrophic cascades  
 just below  $p_c$

ABRUPT TRANSITION (1<sup>st</sup> order)

$$p_c = 2.4554 / \langle k \rangle \quad \text{For a single network } p_c = 1 / \langle k \rangle$$

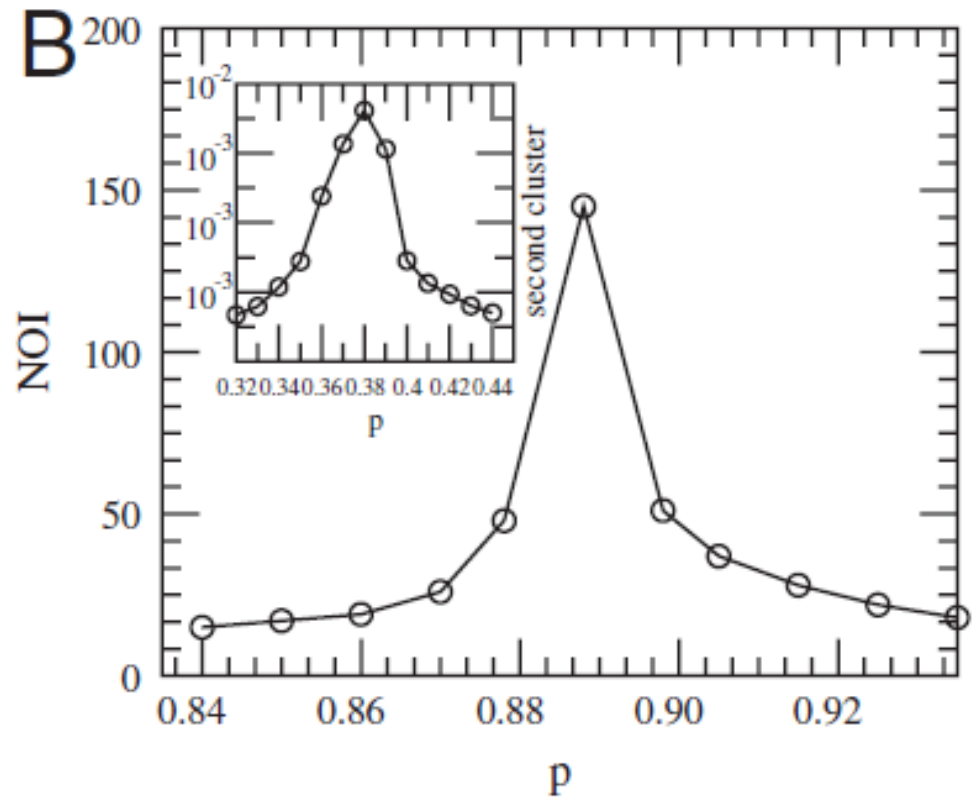
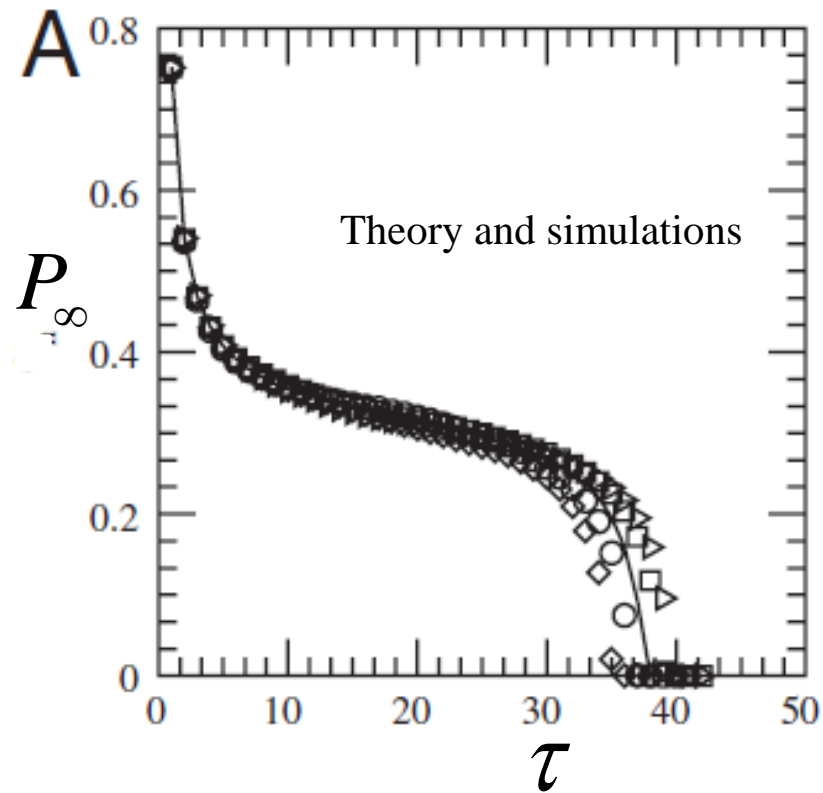
$$\langle k \rangle_{\min} = 2.4554 \text{ for single network } \langle k \rangle_{\min} = 1$$

$$\langle \tau \rangle \sim N^{1/3}$$

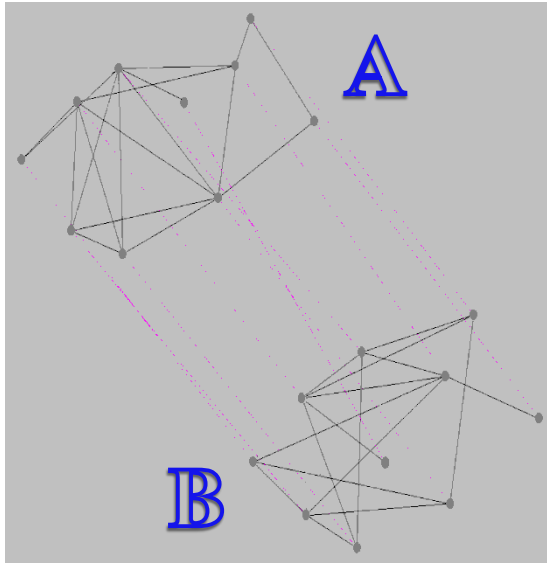
Dong Zhou et al (2013)

# Partial Interdependent Networks

Determining  $p_c$  in simulations:



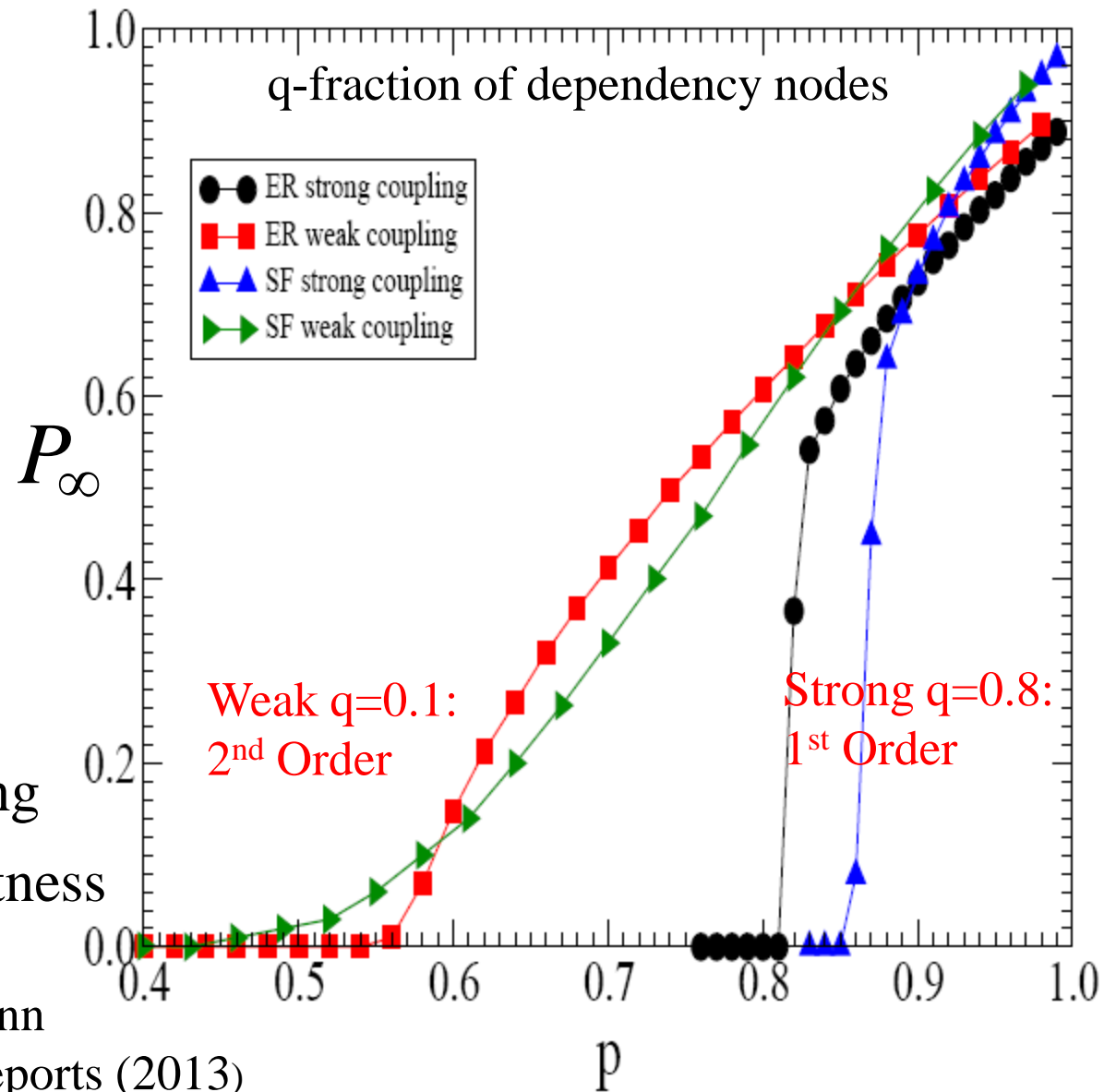
# GENERALIZATION: PARTIAL DEPENDENCE: Theory and Simulations



Parshani, Buldyrev, S.H.  
PRL, **105**, 048701 (2010)

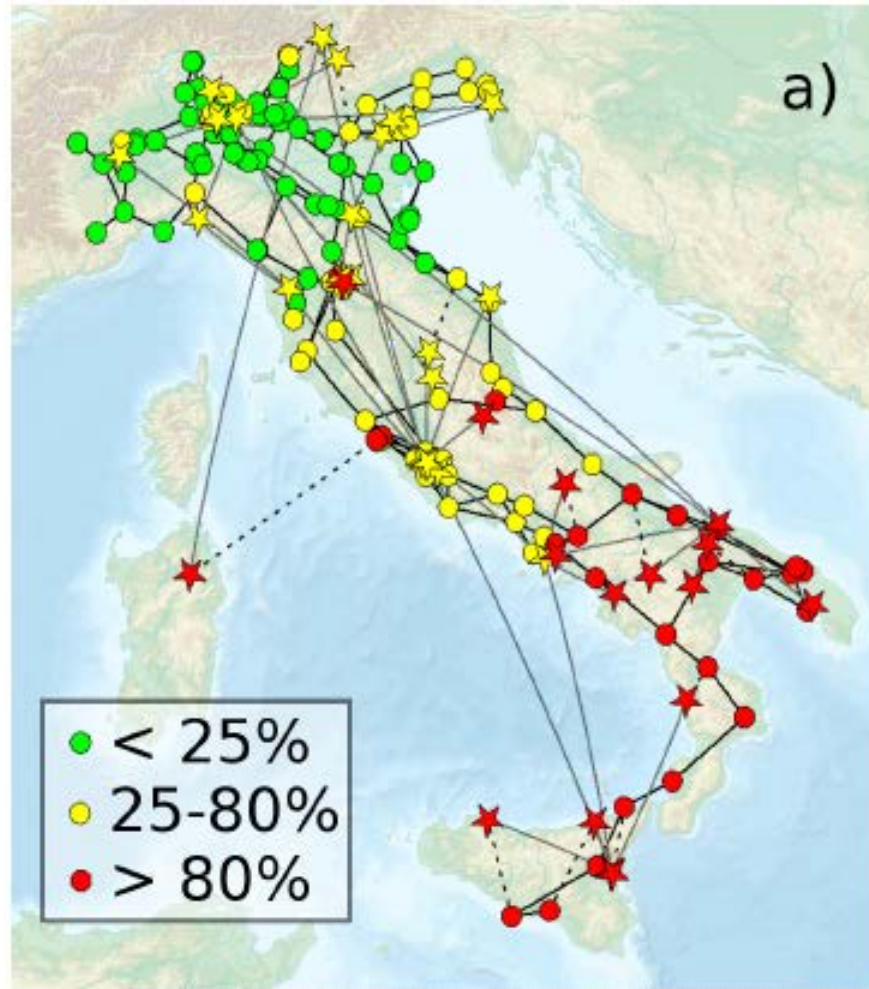
$q_c \cong 0.2$  for random coupling  
 $q_c \rightarrow 0.9$  for optimal robustness

Schneider, Araujo, SH, Herrmann  
[arXiv:1106.3234](https://arxiv.org/abs/1106.3234), Scientific Reports (2013)

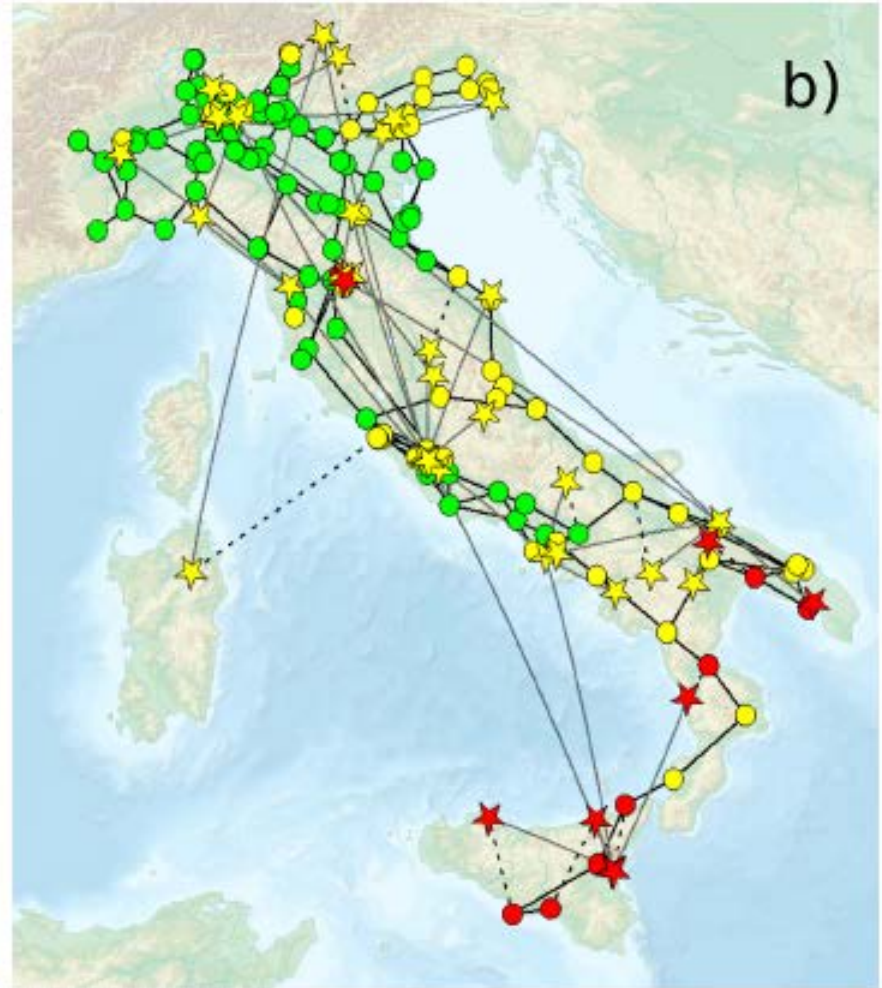


# Designing Robust Coupled Networks: Italy 2003 blackout

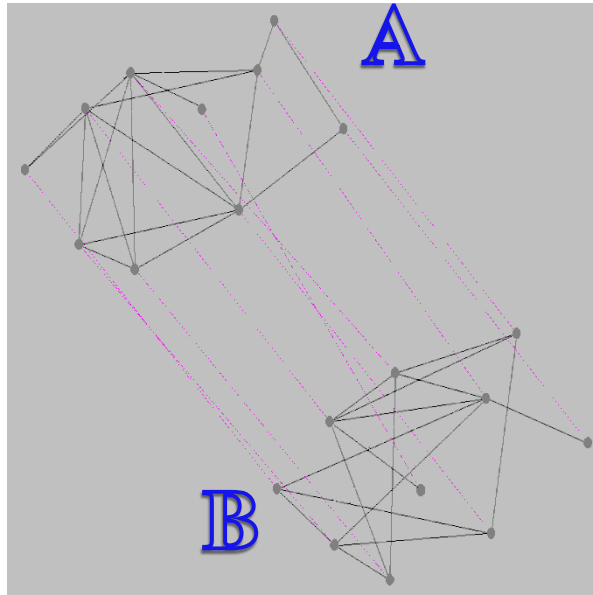
Random interdependencies



Nearly optimal interdependencies



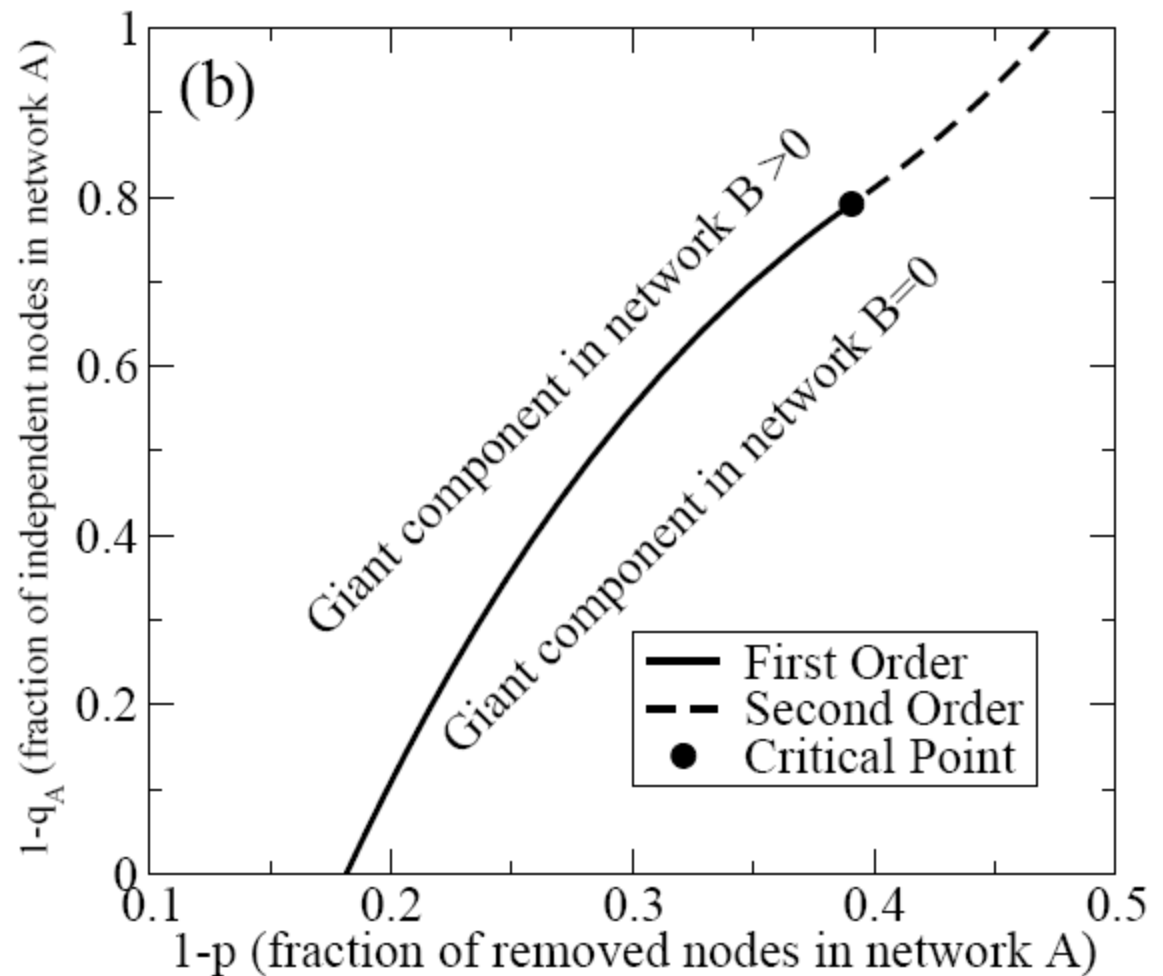
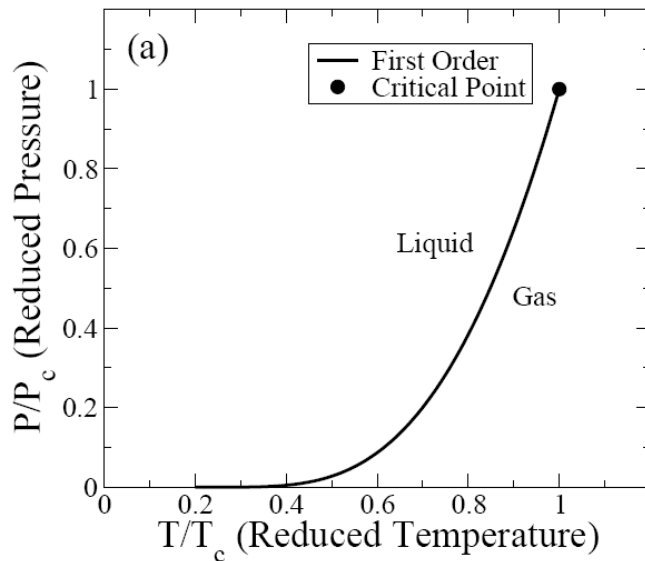




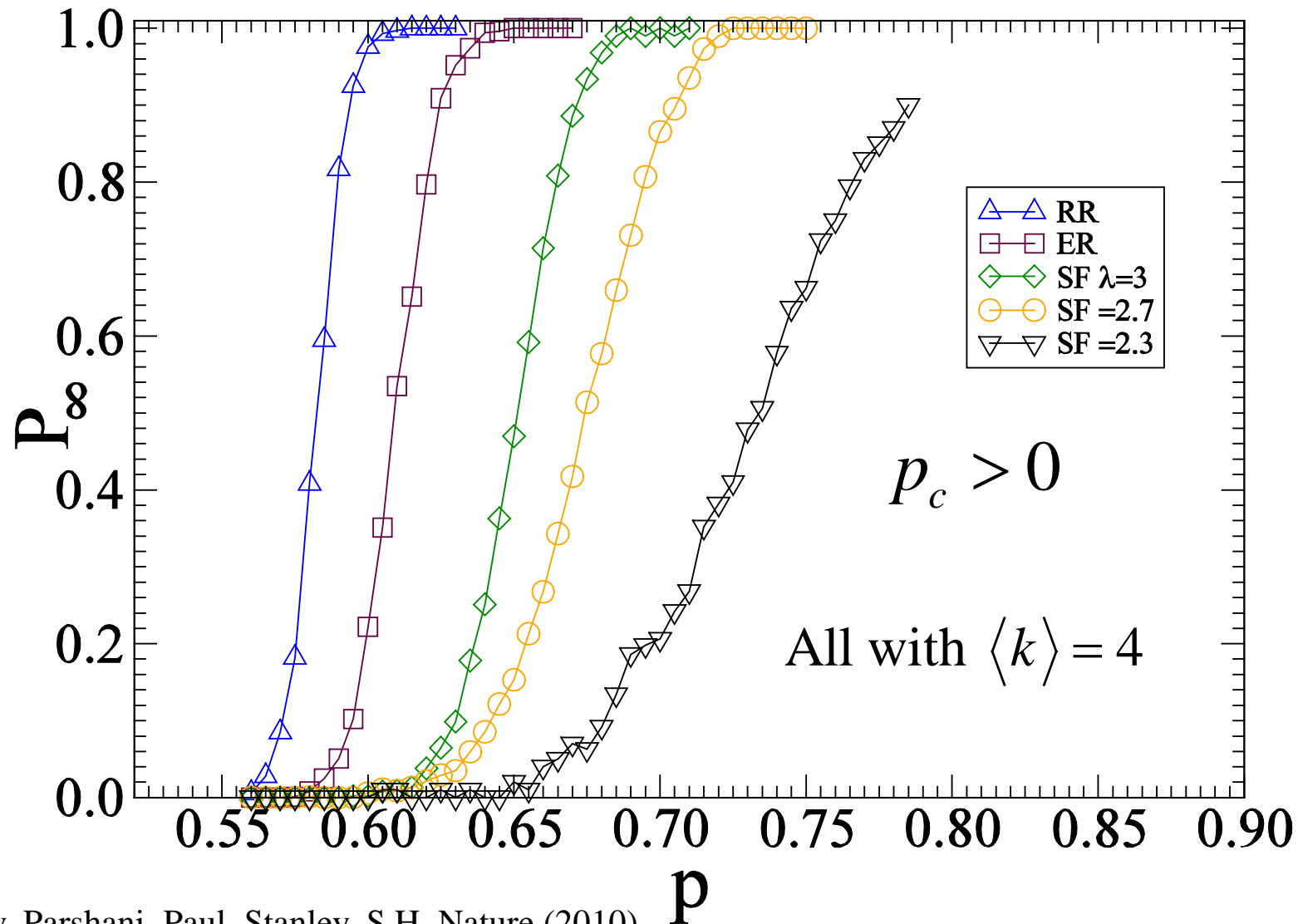
# PARTIAL DEPENDENCE: critical point

Parshani et al  
PRL, **105**, 048701 (2010)

Analogous to **critical point**  
in liquid-gas transition:

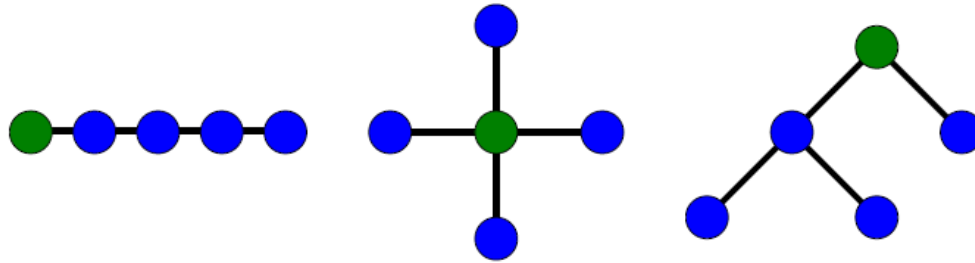


IN **CONTRAST** TO SINGLE NETWORKS, COUPLED NETWORKS  
ARE **MORE VULNERABLE** WHEN DEGREE DIST. IS **BROADER**



# Network of Networks (tree)

n=5



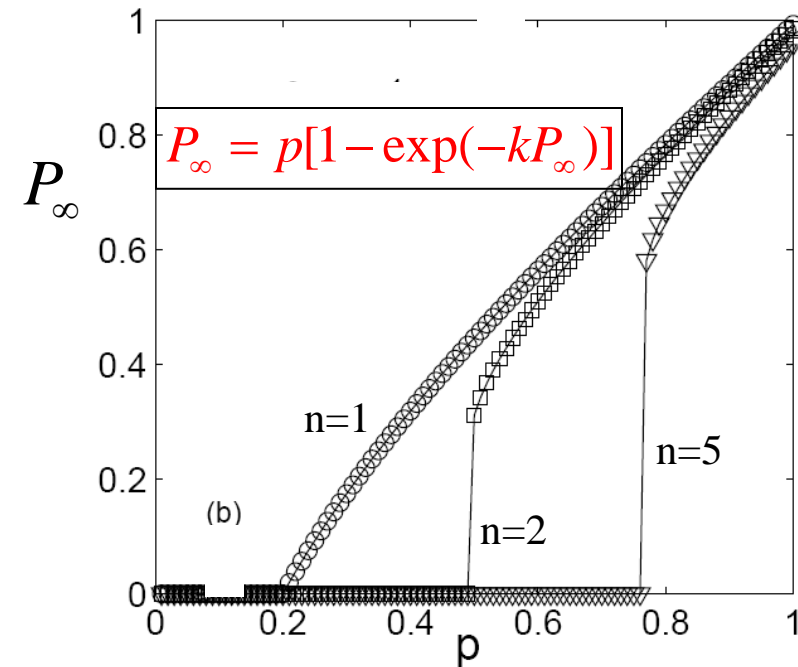
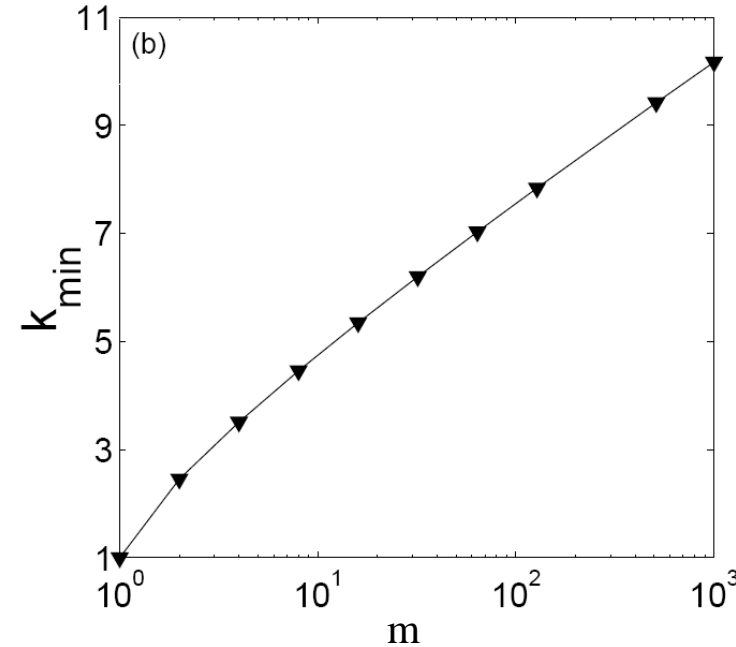
For ER,  $\langle k_i \rangle = k$ , full coupling,  
**ALL** loopless topologies (chain, star, tree):

$$P_{\infty} = p[1 - \exp(-kP_{\infty})]^n$$

n=1 known ER- 2<sup>nd</sup> order

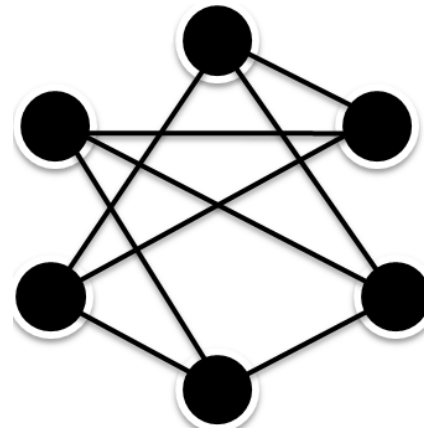
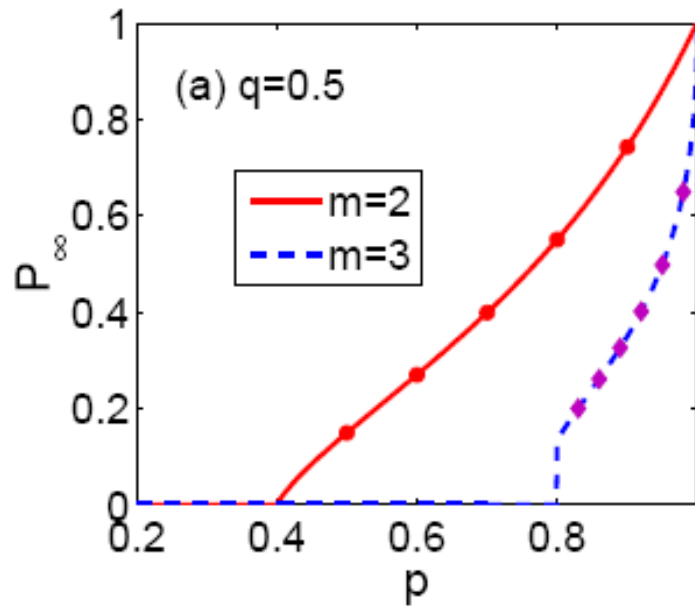
$$p_c = 1 / \langle k \rangle$$

Vulnerability increases significantly with n

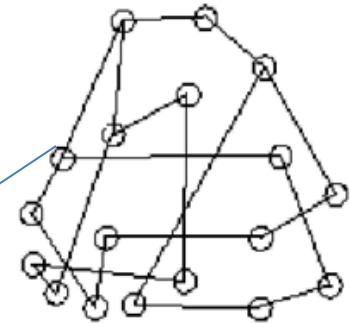


Gao et al PRL (2011)

# Random Regular Network of ER networks



RR,  $m=3$



ER  $\langle k \rangle = 2.2$

$$P_\infty = \frac{p}{2^m} (1 - e^{-\langle k \rangle P_\infty}) [1 - q + \sqrt{(1 - q)^2 + 4qP_\infty}]^m$$

$$p_c = \frac{1}{\langle k \rangle (1 - q)^m}$$

$$q_c = \frac{\langle k \rangle + m - \sqrt{m^2 + 2\langle k \rangle m}}{\langle k \rangle}$$

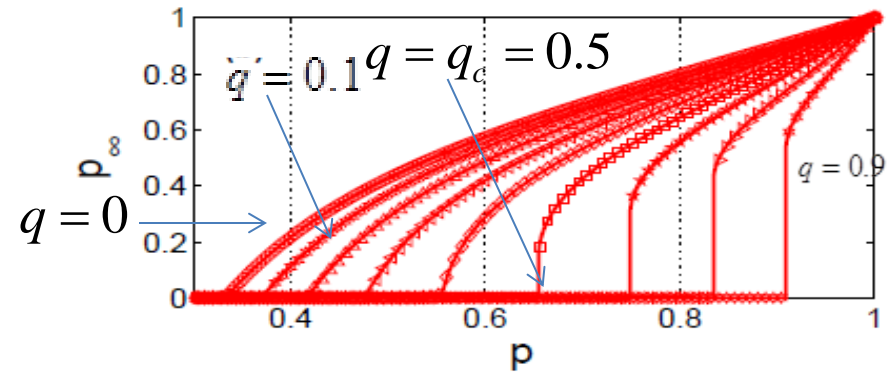
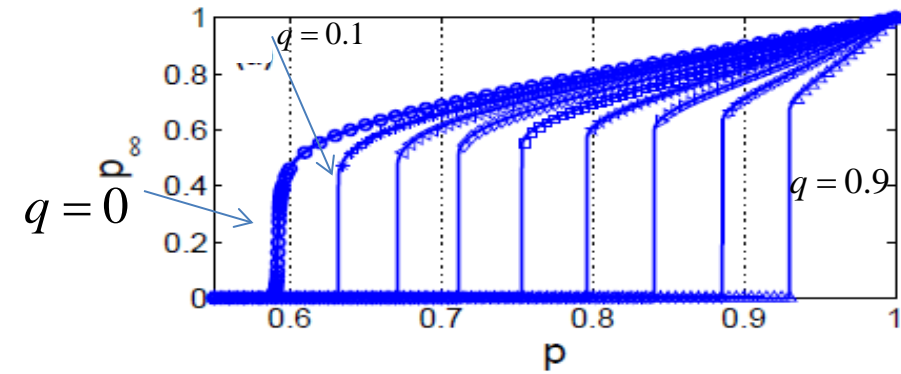
Surprisingly Independent on  $n$ !

For  $m = 0$  OR  $q = 0$   
the single network  
is obtained!

$$P_\infty = p[1 - \exp(-\langle k \rangle P_\infty)]$$



# Spatial embedded compared to random coupled networks when q changes



**EXTREMELY VULNERABLE!!**

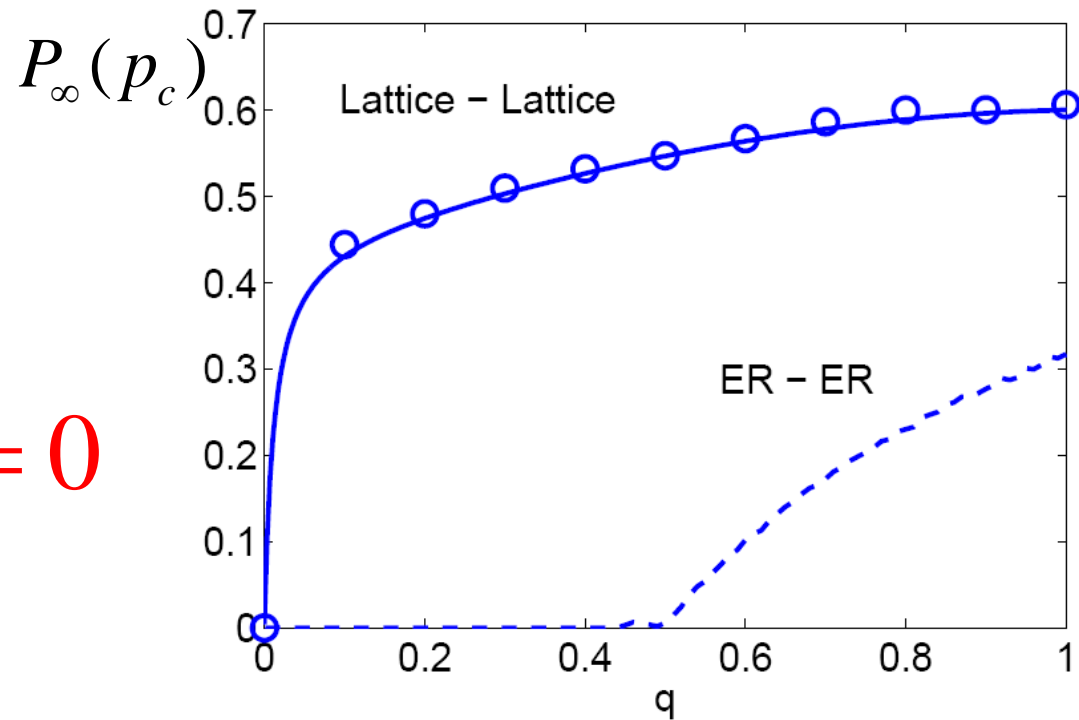
$$1 = p_c q_c P'_\infty(p_c)$$

$$P_\infty \sim (p - p_c)^\beta$$

$$\beta = 5/36 < 1 \text{ for } d=2$$

$$\text{For ER and } d=6, \beta=1$$

$$q_c = 0$$



Bashan et al

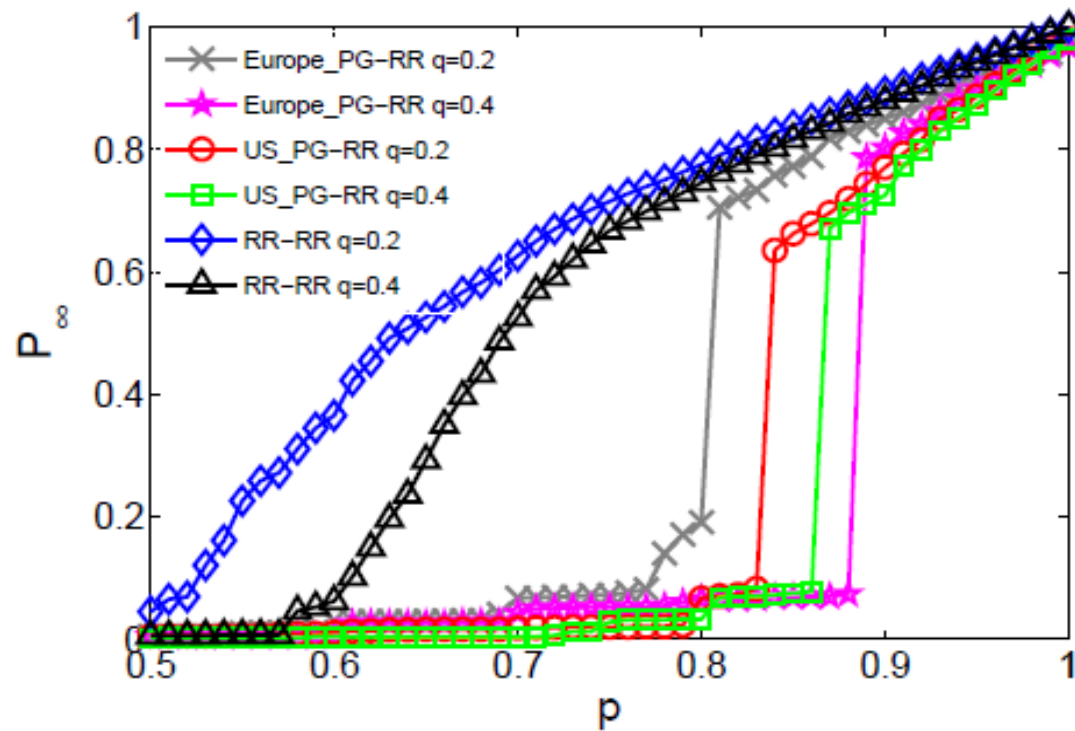
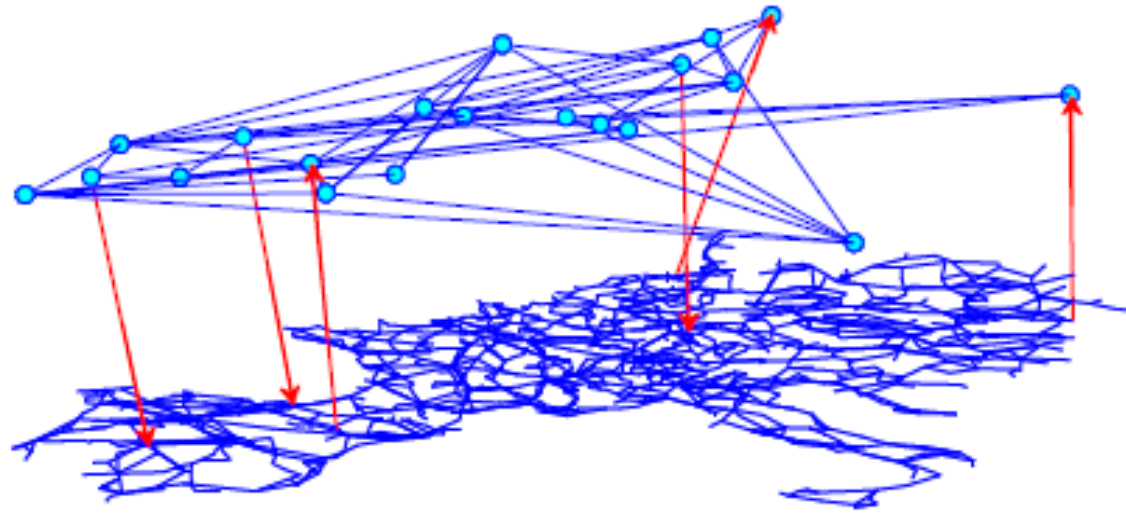
<http://arxiv.org/abs/1206.2062>

Nature Physics, (2013)

**Message: our world is extremely unsafe!-no safe zone!**

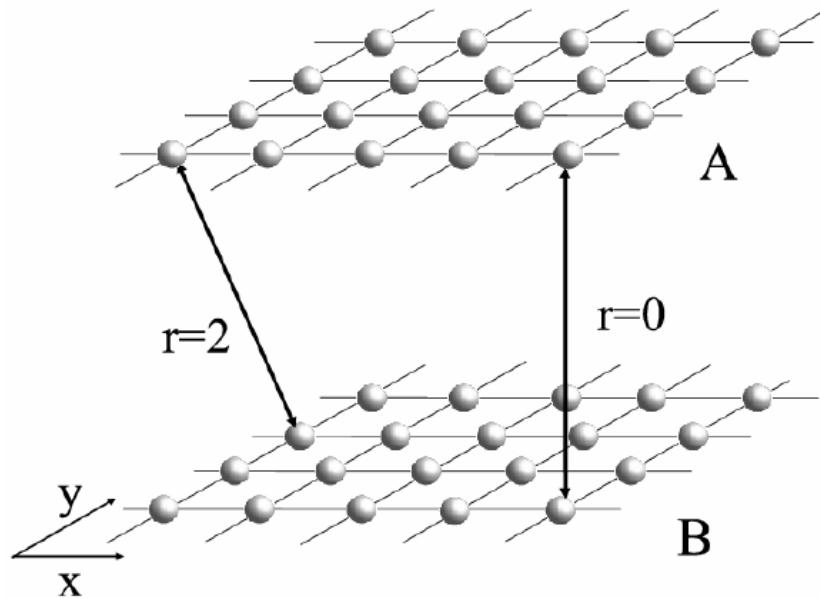
# Test on real spatial embedded coupled networks

US-POWER GRID  
EUROPE POWER GREED



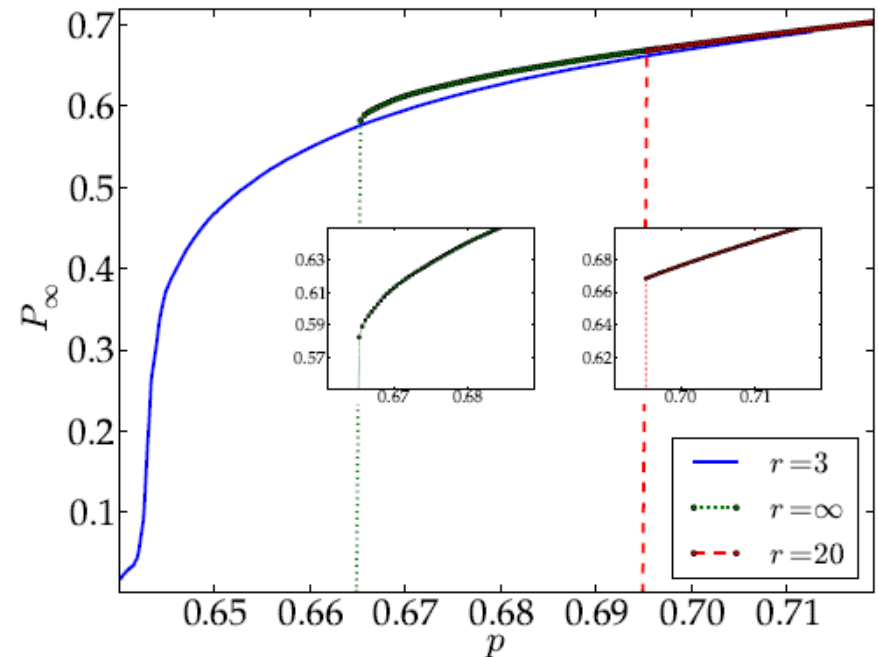
Bashan et al, Nature Physics (2013)  
<http://arxiv.org/abs/1206.2062>

# Interdependent Spatially Embedded Networks



Many networks are spatially embedded:  
Internet, Power grid, Transportation etc

When connectivity links are limited  
in their length---same universality  
class as lattices!

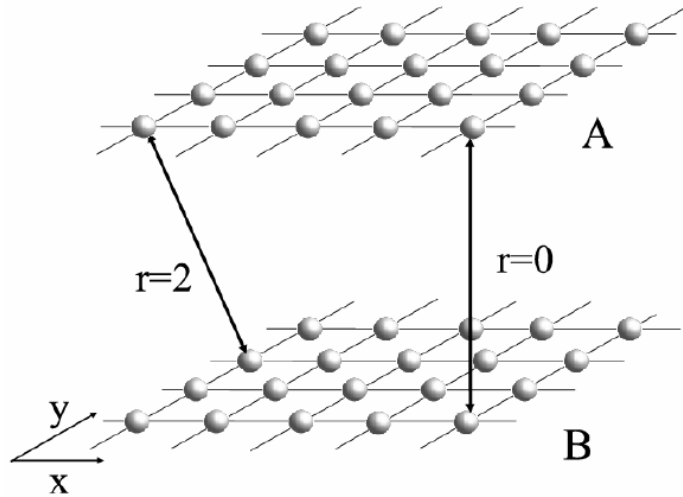


THREE DIFFERENT BEHAVIORS  
DEPENDING ON  $r$

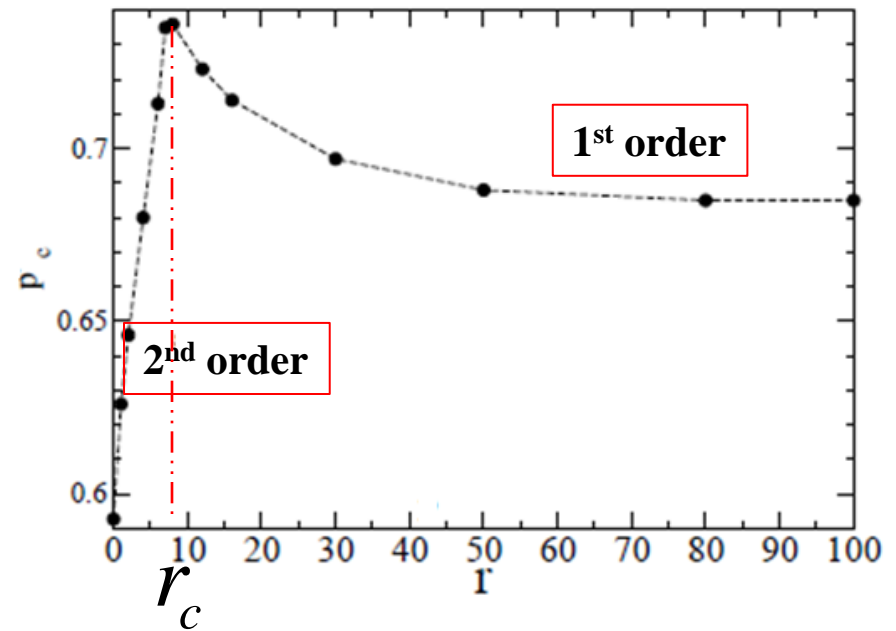
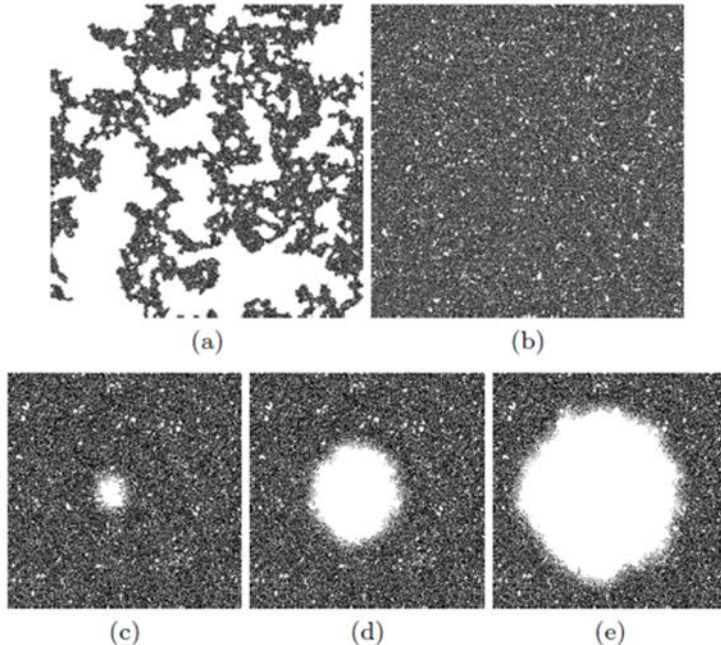
Wei et al, PRL, 108, 228702 (2012)

Bashan et al, <http://arxiv.org/abs/1206.2062>

# Interdependent Spatially Embedded Networks



Many networks are spatially embedded:  
Internet, Power grid, Transportation etc



Wei et al, PRL, 108, 228702 (2012)

Bashan et al, <http://arxiv.org/abs/1206.2062>



# Summary and Conclusions

- **First** statistical physics approach for **robustness of Networks of Interdependent Networks**—**cascading failures**
- New paradigm: **abrupt collapse** compared to **continuous** in single network
- Generalization to “**Network of Networks**”:  $n$  interdependent networks—**50y of graph theory and percolation is only a limited case!**  
Larger  $n$  is more vulnerable—**spatial embedding-extremely unsafe:  $q_c = 0$**

Rich problem: different types of networks and interconnections.

Buldyrev et al., NATURE (2010)  
Parshani et al., PRL (2010)  
Gao et al, PRL (2011)  
Parshani et al, PNAS (2011)  
Wei et al, PRL (2012)  
Gao et al., Nature Phys. (2012)  
Bashan et al, Nature Phys. (2013)

