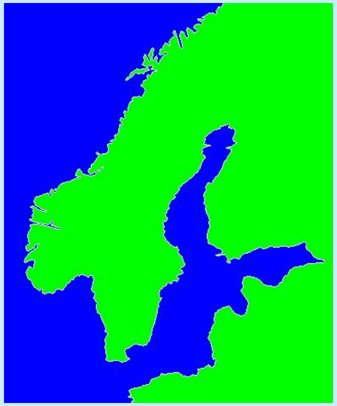
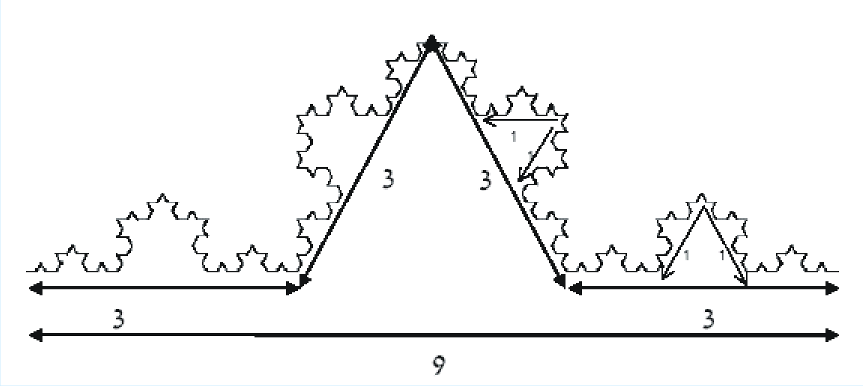
Fractals in Nature
Coastline

What is the length of the coastline for Norway ?



- When measuring with a smaller scale we get a longer coastline
- Thus, no meaning for the length!
- For a scale d , we count steps $N(d)$ along the coast
- The total length is $L = N(d) \cdot d$

Same for Koch curve



For linear scale of size 9 we get a length

$$L = N(d) \cdot d = 1 \cdot 9 = 9$$

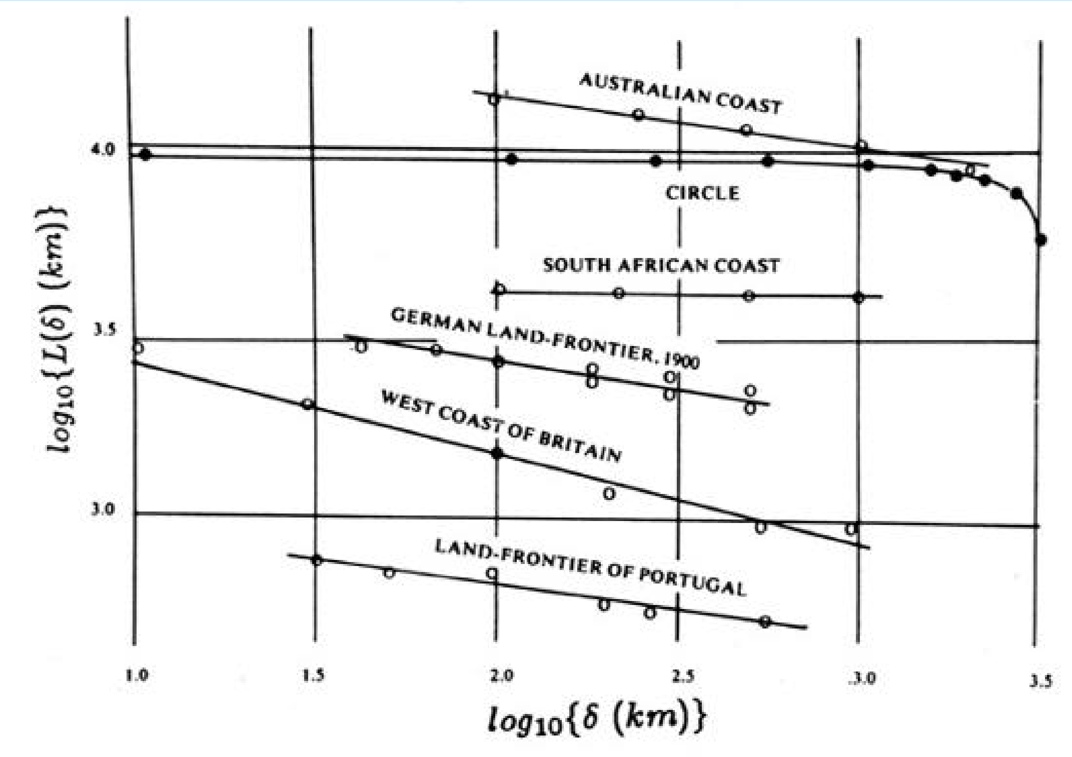
For scale 3: $L = N(d) \cdot d = 4 \cdot 3 = 12$

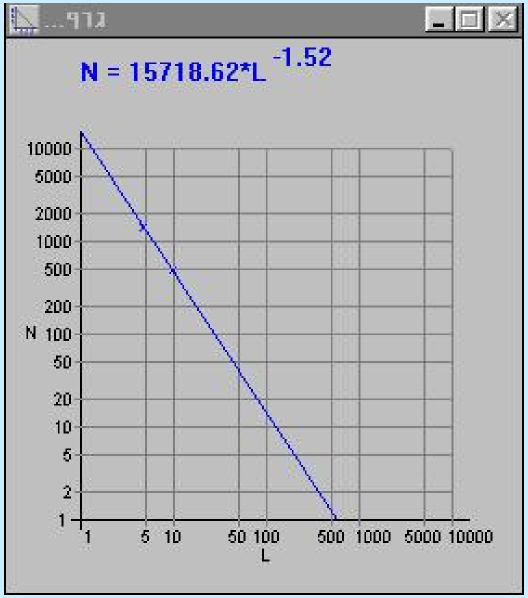
For scale 1: $L = N(d) \cdot d = 16 \cdot 1 = 16$

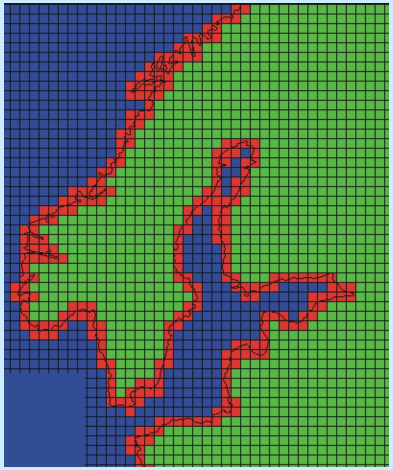
Coastline

From box dimension: the number of boxes needed to cover the coastline

$$N(d) = Ad^{-d_f}$$





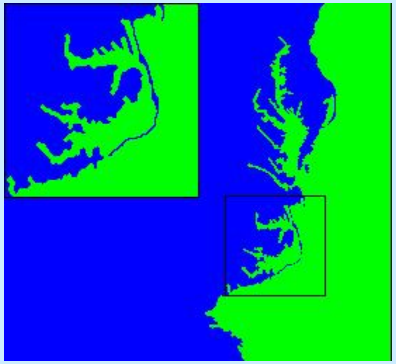


- How many boxes of size e are needed to cover the coast - $N(e)$
- Richardson (1961) found that the length of coast depends on ruler and suggested the scaling law $\equiv$ fractal

Random fractals

In nature there exist many examples of random fractals.

Examples:



Coast lines

Rivers

Mountains

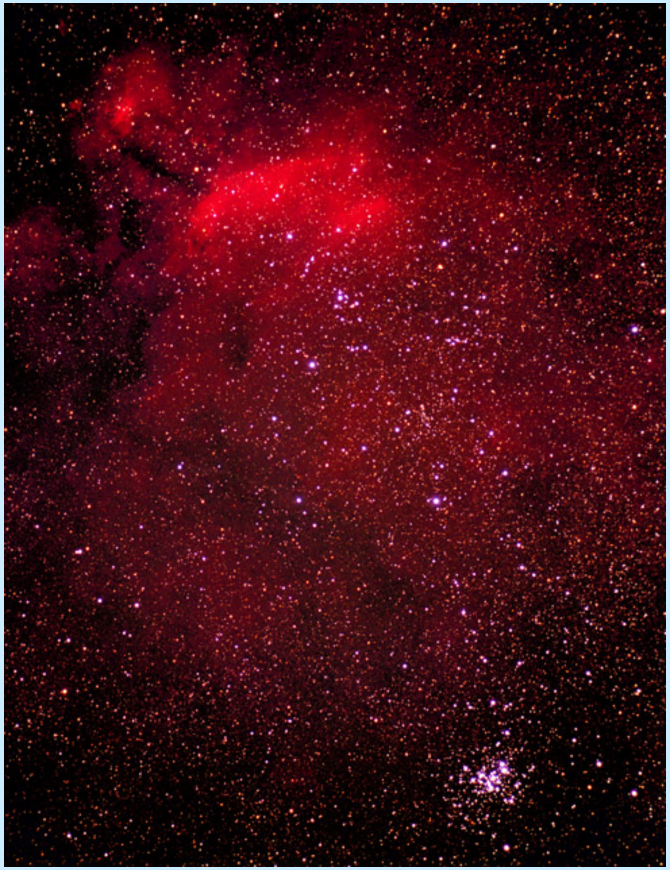
Clouds

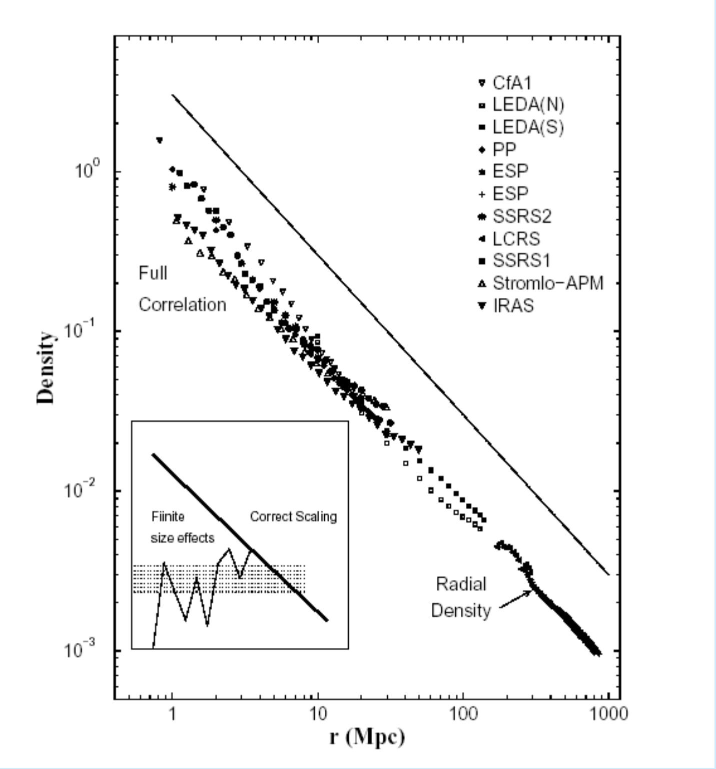
Lightening

Neurons

Galaxies

- From 1900 astronomers found clusters of stars and galaxies
- Novel results (Pietronero, 1992) show fractal features for the galaxies
- For random distribution of stars or galaxies the fractal dimension would be $d_f = 3$.
- Observations for length scales up to 20 Mpc (1pc$\equiv 3.08 \cdot 10^3$ km) yield $d_f \cong 1.23 \pm 0.04$

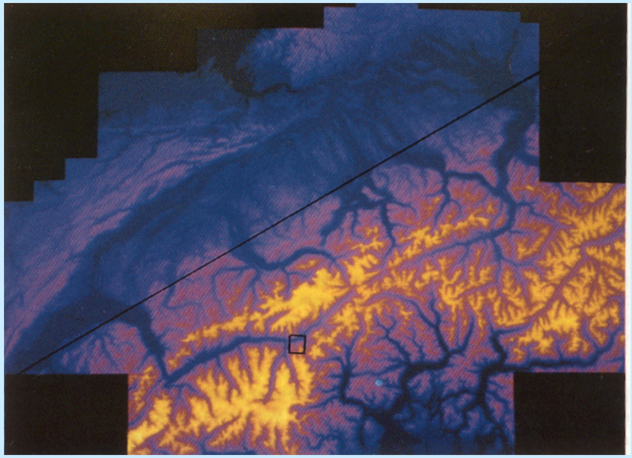




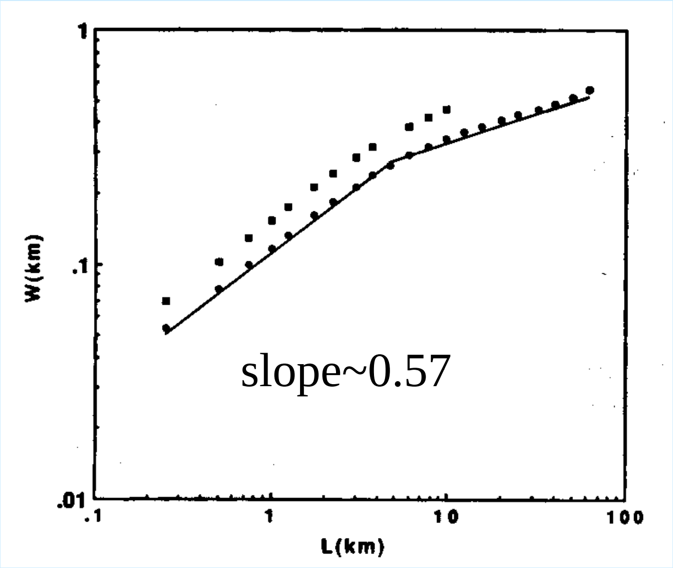
Fractal aspects of the Swiss landscape

Giovanni Dietler and Yi-Cheng Zhang

Physica A 191 (1992) 213-219



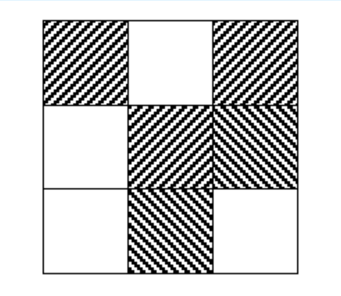
$slope \approx 0.57$



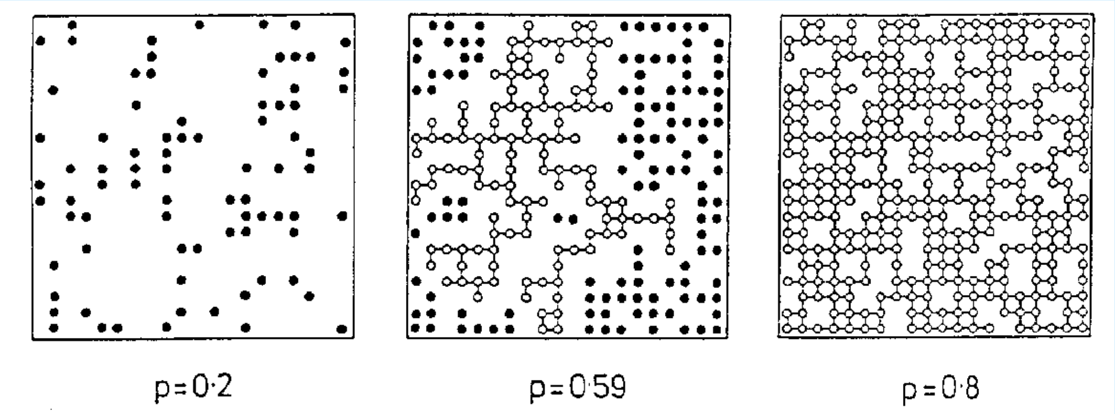
Percolation

✓Model for disordered media

P=1/2



✓Each site is occupied with probability p and empty with probability 1-p



✓For low p – small clusters
✓For large p – big clusters – Infinite cluster
✓At $p=p_c$ a transition from small clusters to infinite clusters
✓Occupied and empty sites can represent different physical properties, e.g.
occupied – conductors
empty – isolators
✓Current can flow only on conductors

below p_c – isolator
above p_c – conductor } Isolator-conductor phase transition

✓p_c – called "critical concentration" – above which current cannot flow
✓p_c – called also "percolation threshold"

Percolation

More examples

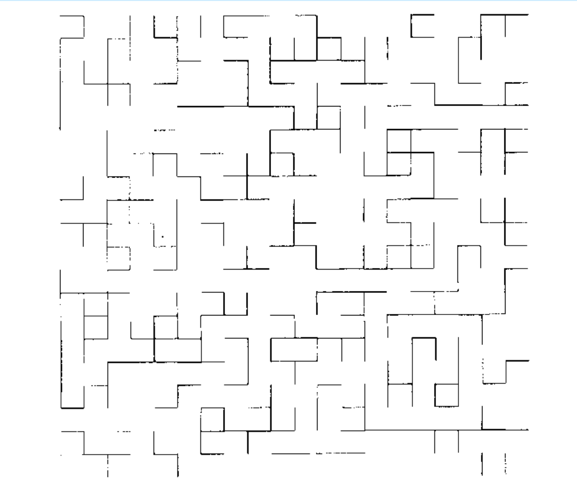
✓Occupied sites – superconductors
Empty sites - conductors
} Superconductor – conductor phase transition (at p_c)

✓Occupied sites – magnets
Empty sites - paramagnets
} Magnet - paramagnet phase transition (at p_c)

✓Occupied sites – working computers
Empty sites – damaged computers
} Internet network phase transition

✓Comparison with thermal phase transition

solid-liquid
critical temperature T_c
below T_c – order (infinite cluster)
above T_c – disorder (small clusters)

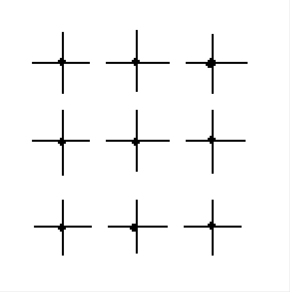


Bond Percolation

✓Bonds are occupied randomly with probability p
✓At p_c an infinite cluster of bonds appears
✓Model for random resistor network: bonds are cut randomly

Bond Percolation - Examples

Chemistry - polymerization



- ✓Branching molecules can perform larger molecules by activating more and more bonds
- ✓Assume that probability to activate a bond is p
  - below p_c – small macromolecules
  - above p_c – large macromolecules (system size)
- ✓Called sol-gel transition

Gel – infinite cluster – elastic (like food gels) – above p_c
Sol – viscous fluid – below p_c

- ✓Example – boiled egg
  heating – activates more bonds between molecules

Biology – epidemic spreading

- ✓Epidemic starts with a single sick person that can infect its neighbors with probability p (per unit time)
- ✓Neighbors can infect their neighbors
- ✓If p is small the epidemic stops. Above p_c the epidemic spreads to large populations
- ✓Model also for fire spreading in a forest

✓Percolation aspects are important in many systems in Nature: amorphous and porous materials (e.g. rocks), branched polymers, fragmentation, galaxies structure, earthquakes, anomalous properties of water, simulations of oil recovery from porous rocks.

Percolation Threshold

✓Site and bond percolation can be defined for all lattices and for all d

✓In general a bond has more neighbors than a site

Example: square lattice site has 4 neighbors

bond has 6 neighbors

Thus, big clusters of bonds are easier generated than for sites

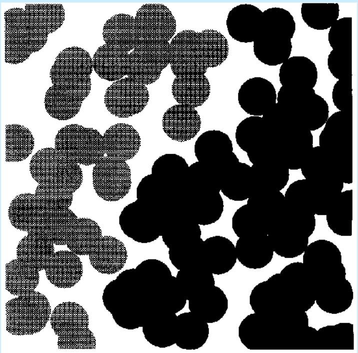
$\Rightarrow p_c$ *for bonds* $<$ p_c *for sites* for the same lattice

✓Example:

$$\left.\begin{array}{ll} p_c = 1/2 & \text{for bond percolation} \\ p_c = 0.593 & \text{for site percolation} \end{array}\right\} \text{on square lattice}$$

Percolation		Lattice
bond - p_c	**site - p_c**	
$2\sin p/18$	$1/2$	Triangle
$1/2$	0.5927	Square
0.2488	0.3116	Cubic

Continuum Percolation



- ✓Natural example – continuum percolation
- ✓Two components not on a lattice
- ✓Example: take a conducting plate
  make circular holes randomly

- ✓Called: Swiss Cheese Model
- ✓P_c=0.312±0.005 for d=2; p_c=0.034 for d=3
  above p_c – conductor
  below p_c – insulator
- ✓Model for porous materials

Historical remarks

- ✓First work on percolation – Flory +Stockmayer (1941-1943)
  studied gelation or polymerization

- ✓Name percolation – Broadbent and Hammersley (1957)
  studied flow of liquid in porous media
  presented several concepts in percolation

- ✓The developments in phase transition (1960's), series expansion (Domb), renormalization group, scaling theory and universality by Wilson (Nobel Prize), Fisher and Kadanoff – helped to develop percolation theory and understand the percolation as a critical phenomena

- ✓Fractal concept (Mandelbrot, 1977) – new tools (fractal geometry) together with computer development $\Rightarrow$ pushed forward the percolation theory

- ✓Still – many open questions exist !