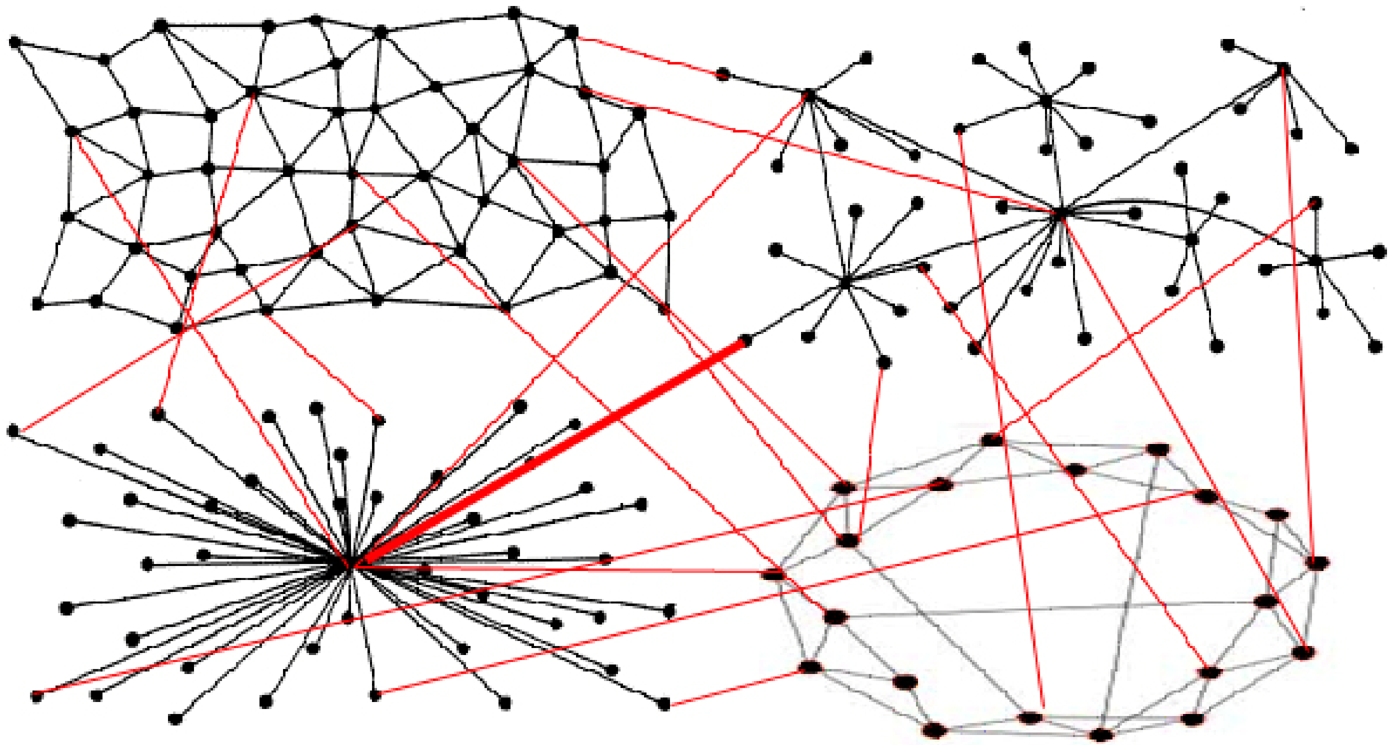
Percolation of Network of Networks

Work with:
S. Buldyrev (NY)
R. Parshani (BIU)
G. Paul (BU)
Jianxi Gao (BIU)
H. E. Stanley (BU)
Nature, 464, 1025 (2010)
PRL ,105, 0484 (2010)
PNAS, 108, 1007 (2011)
Gao et. al. arXiv:1010.5829



Recent results:
Jia Shao (BU)
Amir Bashan (BIU)
Xuqing Huang (BU)
Yanqing Hu (BIU)

Electric grid
Communication
Transport….

Cascading disaster

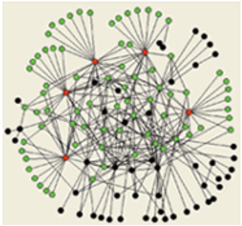
Shlomo Havlin
Bar-Ilan University
Israel

Two types of links:
1. Connectivity
2. Dependency

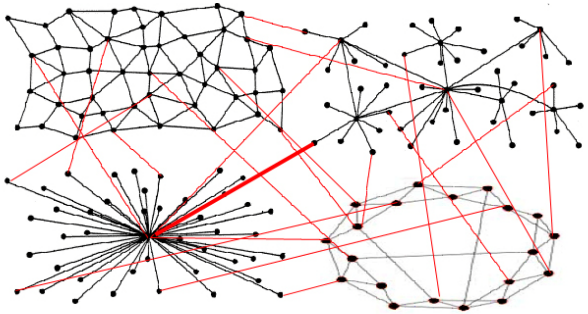
Raissa D'sousa-same type

Interdependent Networks

- Until now studies focused on the case of a single network which is isolated AND does not interact or influenced by other systems.

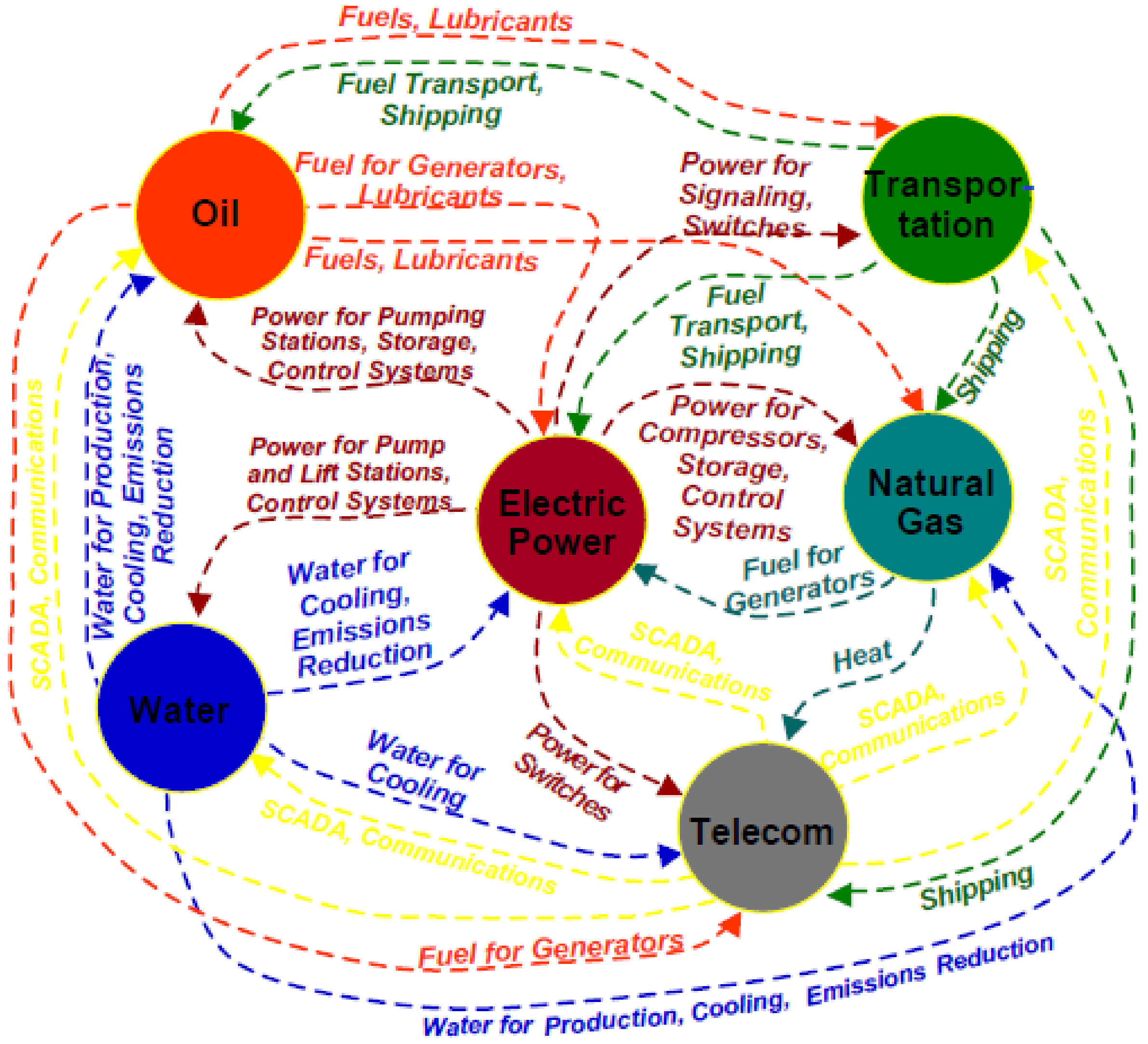


- Isolated systems rarely occur in nature or in technology -- analogous to non-interacting particles (molecules, spins).



- Results for interacting networks are strikingly different from those of single networks.

How interdependent are infrastructures?

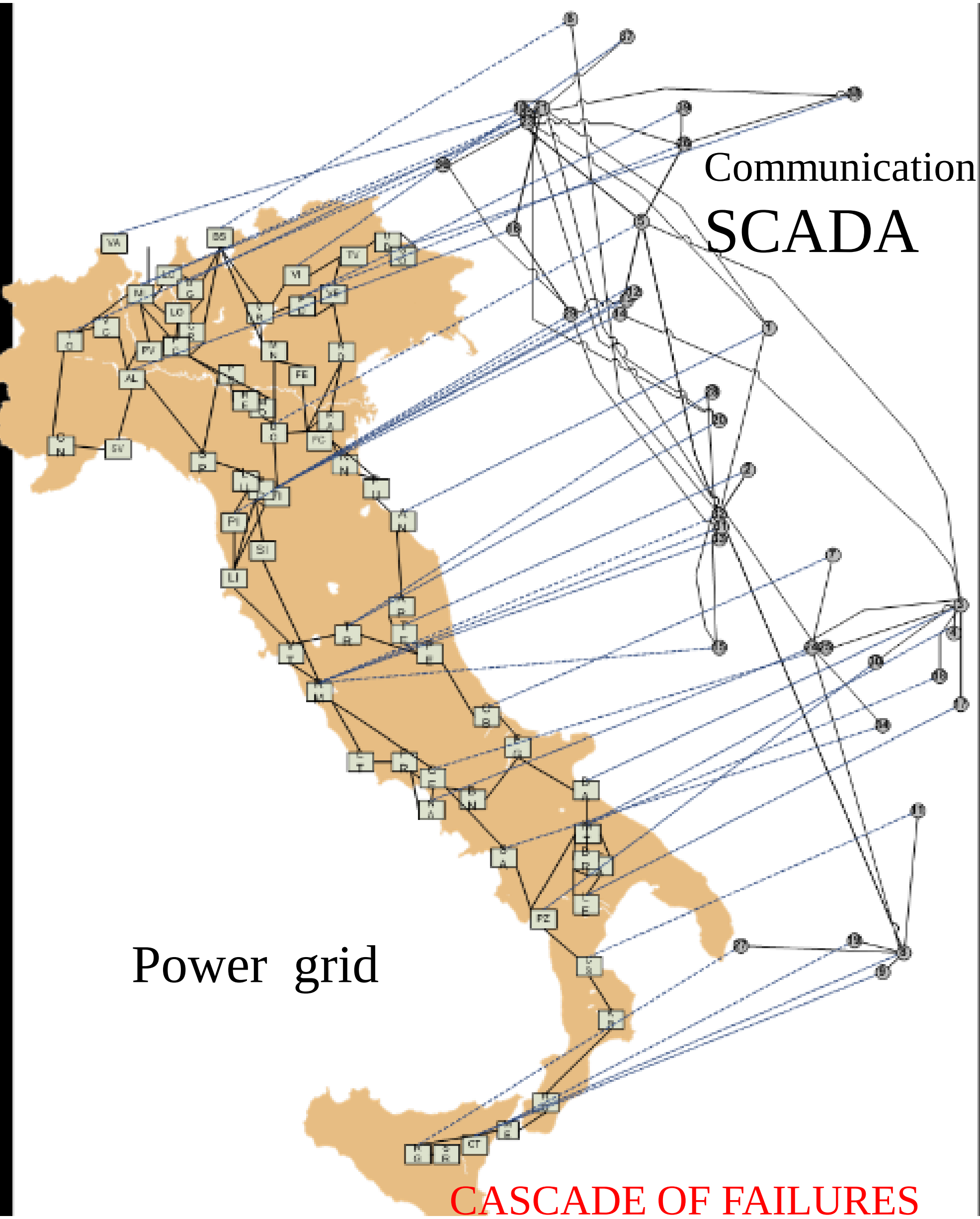


Peerenboom, Fisher, and Whitfield, 2001

Blackout in Italy (28 September 2003)

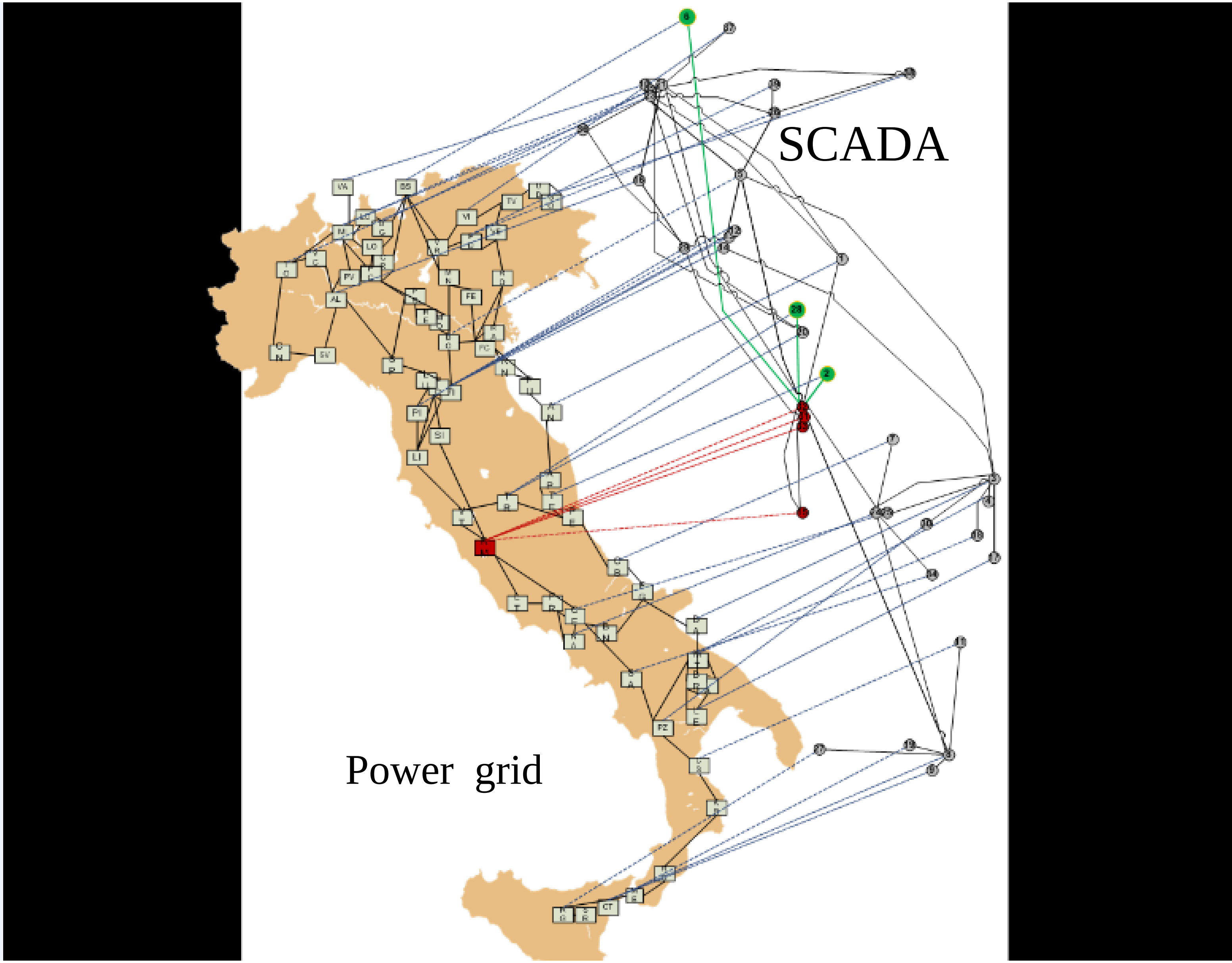
Cyber Attacks- CNN Simulation (2010)

Rosato et al Int. J. of Crit. Infrastruct. 4, 63 (2008)



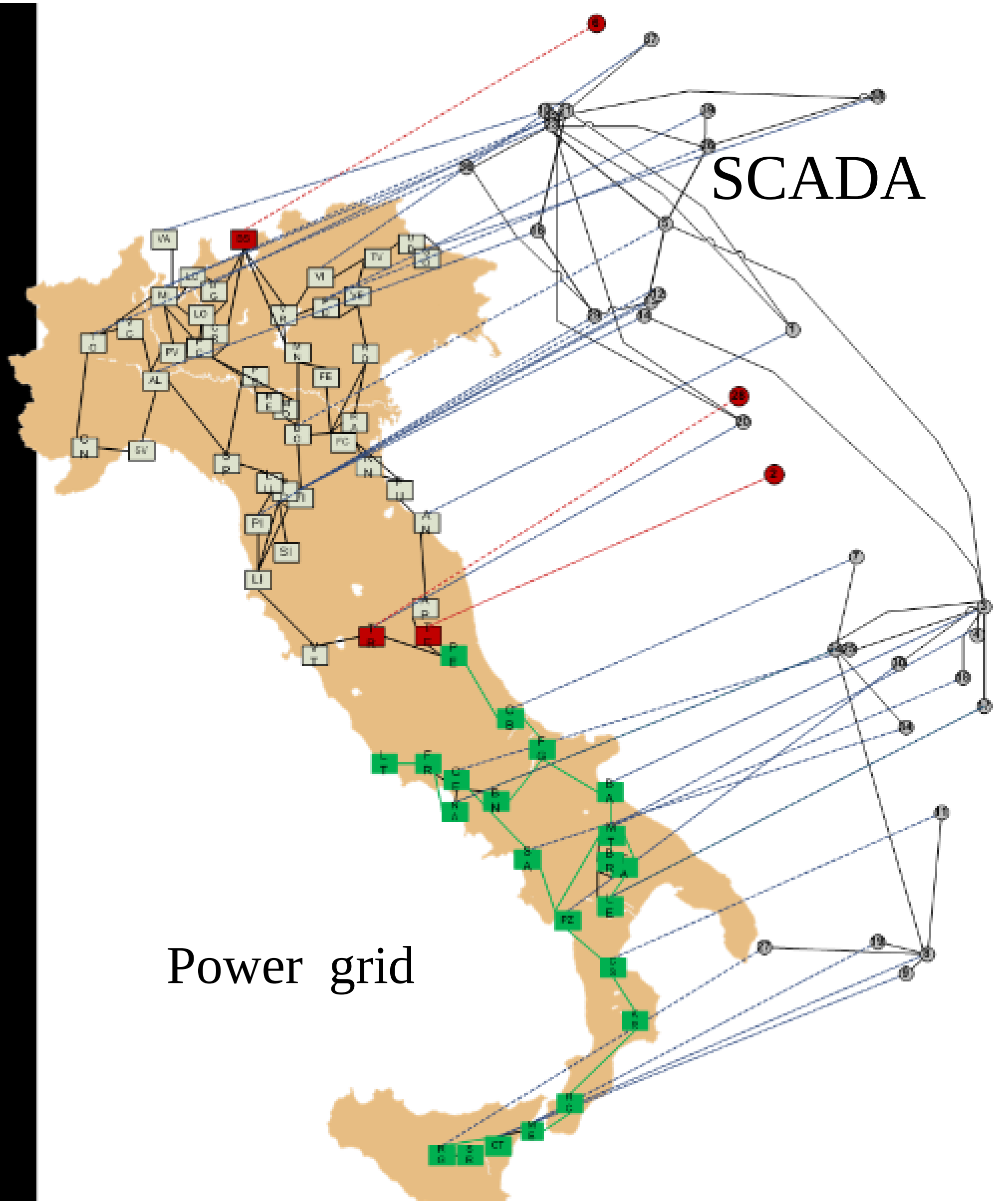
Railway network, health care systems, financial services, communication systems

Blackout in Italy (28 September 2003)

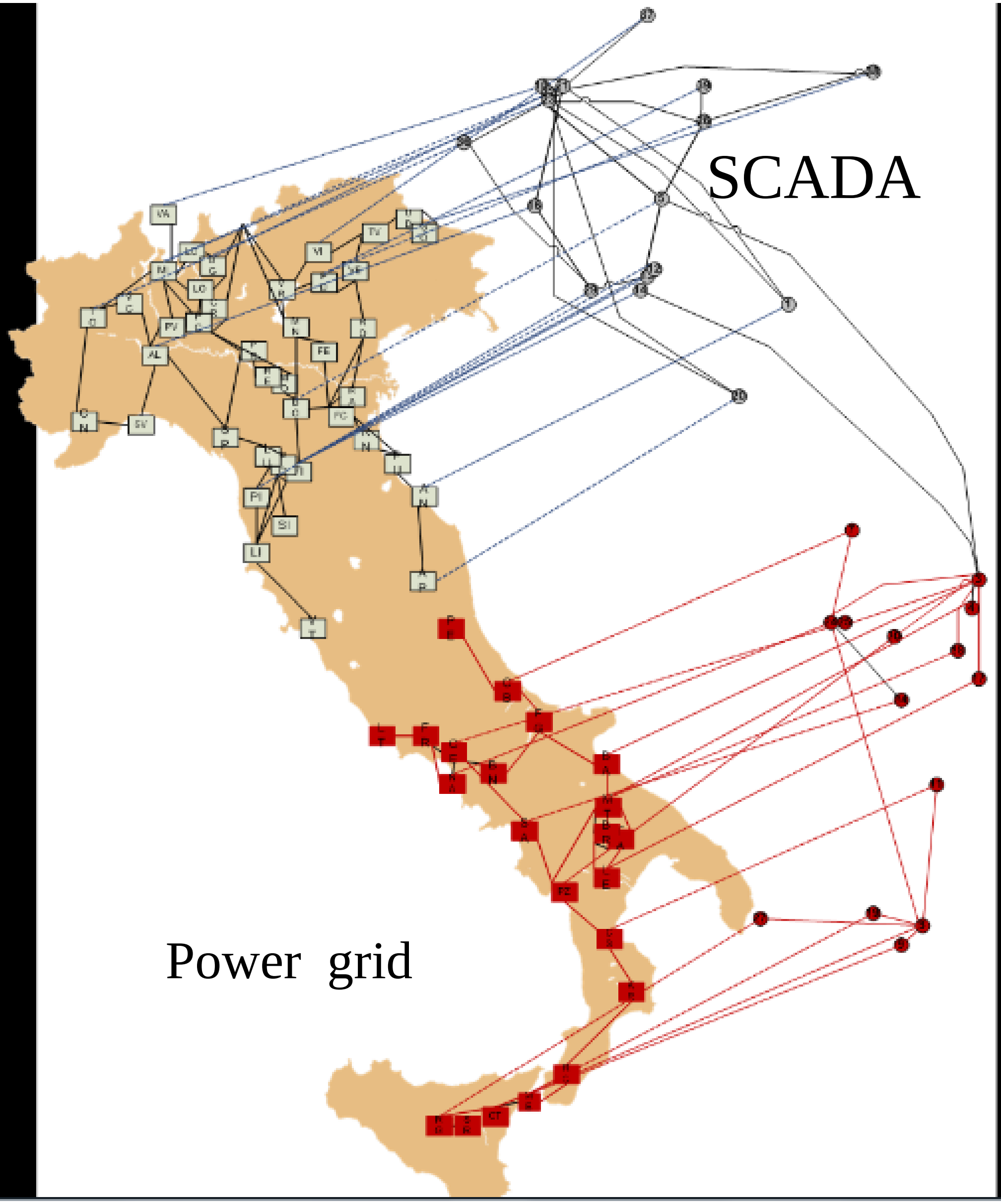


SCADA=Supervisory Control And Data Acquisition

Blackout in Italy (28 September 2003)

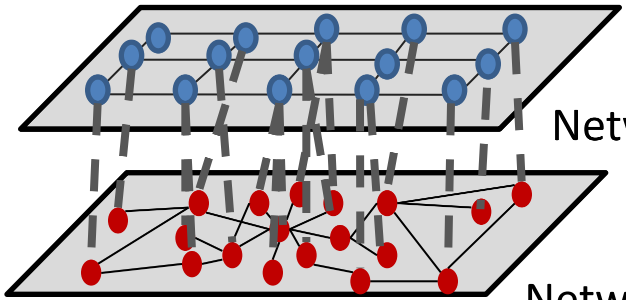


Blackout in Italy (28 September 2003)



Further Examples of Interdependent Networks

Appear in all aspects of life, nature and technology



- *Economy*: Networks of banks, insurance companies, and firms interact and depend on each other.

- *Physiology*: The human body can be regarded as inter-dependent networks. For example, the cardio-vascular network system, the respiratory system, the brain network, and the nervous system all depend on each other.

- *Transportation* : Railway networks, airline networks and other transportation systems are interdependent.

Critical Breakdown Threshold of Interdependent Networks

Failure in network A
causes failure in B → causes further failure in A …..CASCADES
What are the critical percolation thresholds for such interdependent networks?
What are the sizes of cascade failures?

Buldyrev, Parshani, Paul, Stanley, S.H., Nature, (2010); Parshani, Buldyrev, S.H. , Phys. Rev. Lett., (2010)

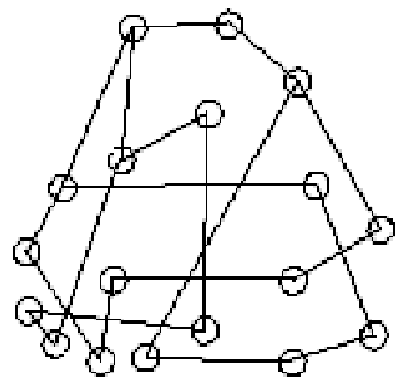
Robustness of a single network: Percolation

Remove randomly (or targeted) a fraction $1-p$ nodes

P_∞ Size of the largest connected component (cluster)
ORDER PARAMETER

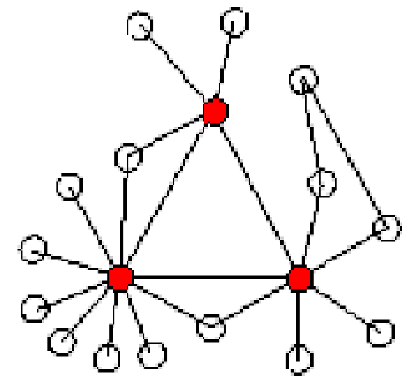
p_c Breakdown threshold

Exponential (ER)



$$P(k) = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

Scale-free (SF)

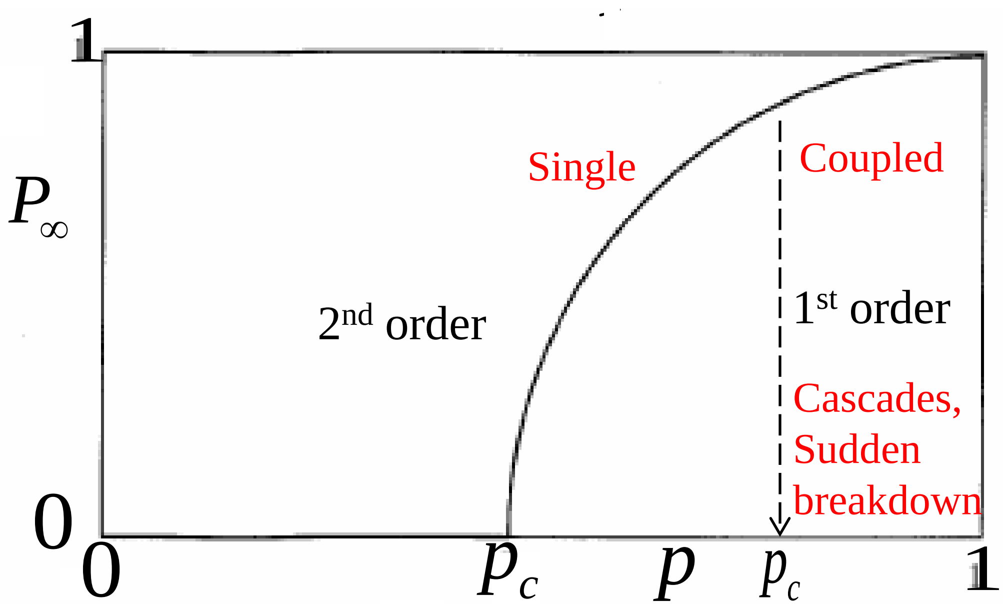


$$P(k) = \begin{cases} ck^{-\lambda} & m \le k \le K \\ 0 & otherwise \end{cases}$$

FOR RANDOM REMOVAL

ER: $p_c = 1/\langle k \rangle$ 2nd order

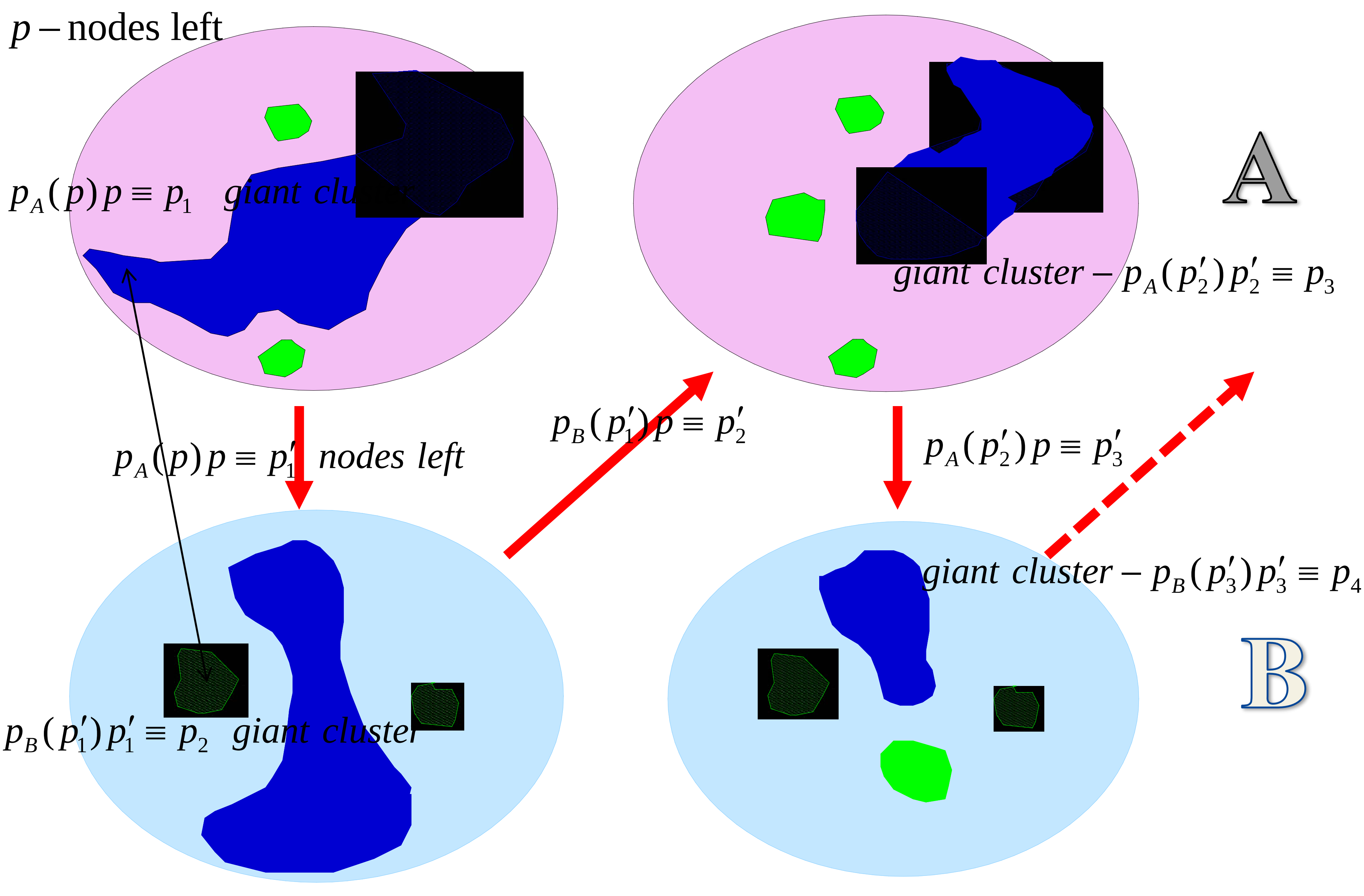
SF: $p_c = 0 \rightarrow$ very robust



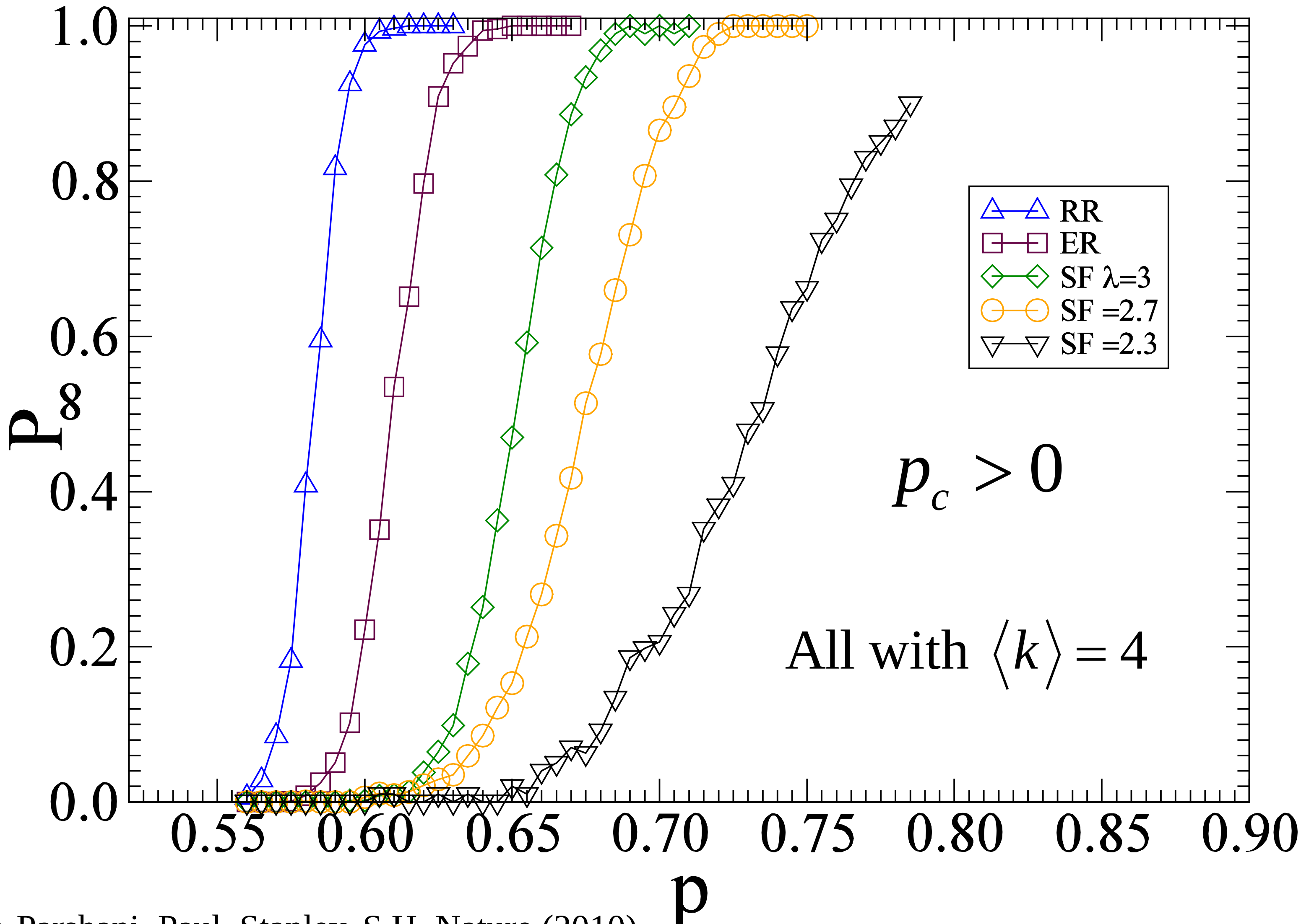
In contrast--in coupled networks:

1. First Order-highly vulnerable
2. Cascading Failures
3. Broader degree-less robust!

RANDOM REMOVAL – PERCOLATION FRAMEWORK



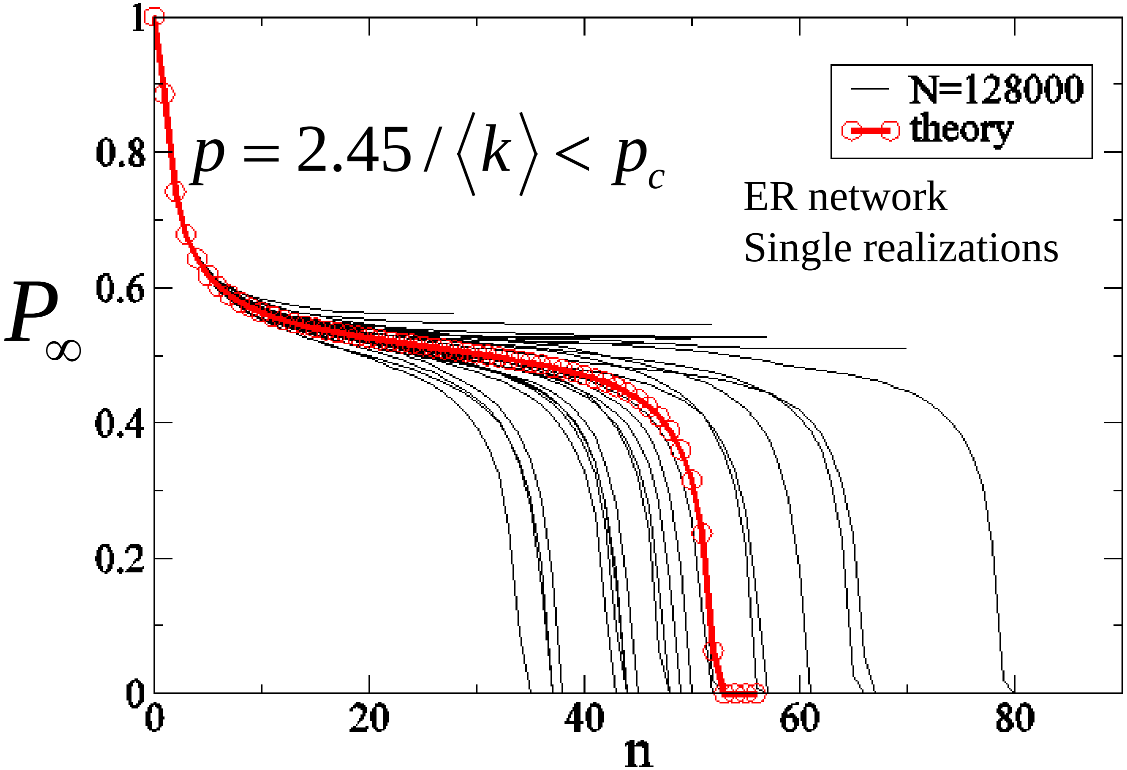
IN CONTRAST TO SINGLE NETWORKS, COUPLED NETWORKS ARE MORE VULNERABLE WHEN DEGREE DIST. IS BROADER



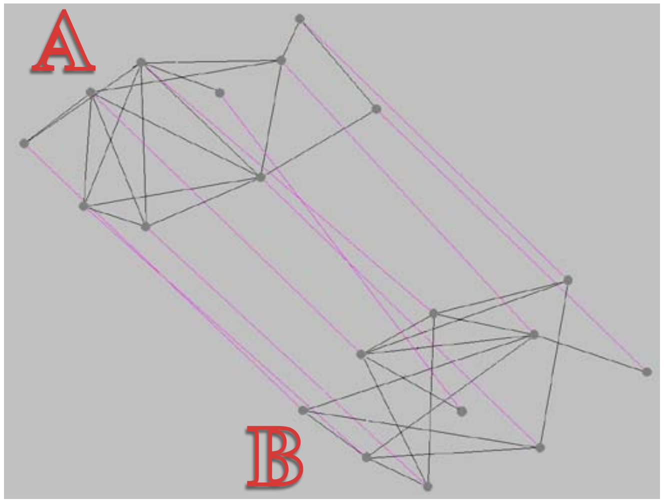
Buldyrev, Parshani, Paul, Stanley, S.H. Nature (2010)

RESULTS: THEORY and SIMULATIONS: ER Networks

P_∞ after n-cascades of failures



Removing 1-p nodes in A



Catastrophic cascades just below p_c

Theory: iterations of the mutual giant component-using generating functions

Single networks

Second order transition

$$p_c = 1 / \langle k \rangle$$

$$\langle k \rangle_{\min} = 1;$$

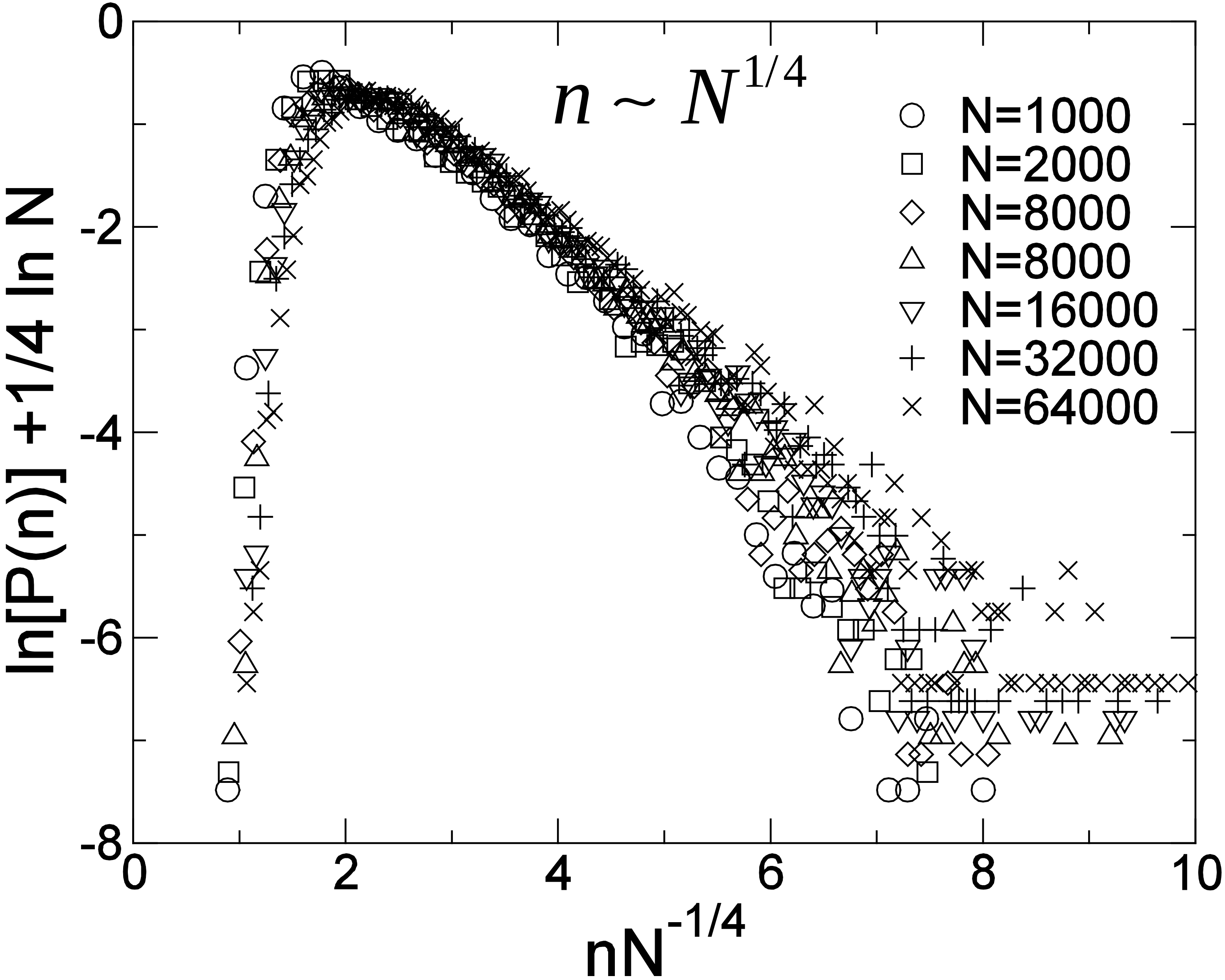
Coupled networks

FIRST ORDER TRANSITION

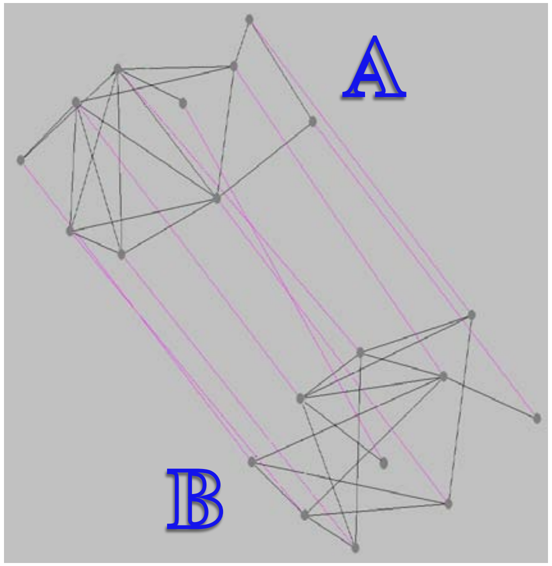
$$p_c = 2.4554 / \langle k \rangle$$

$$\langle k \rangle_{\min} = 2.4554$$

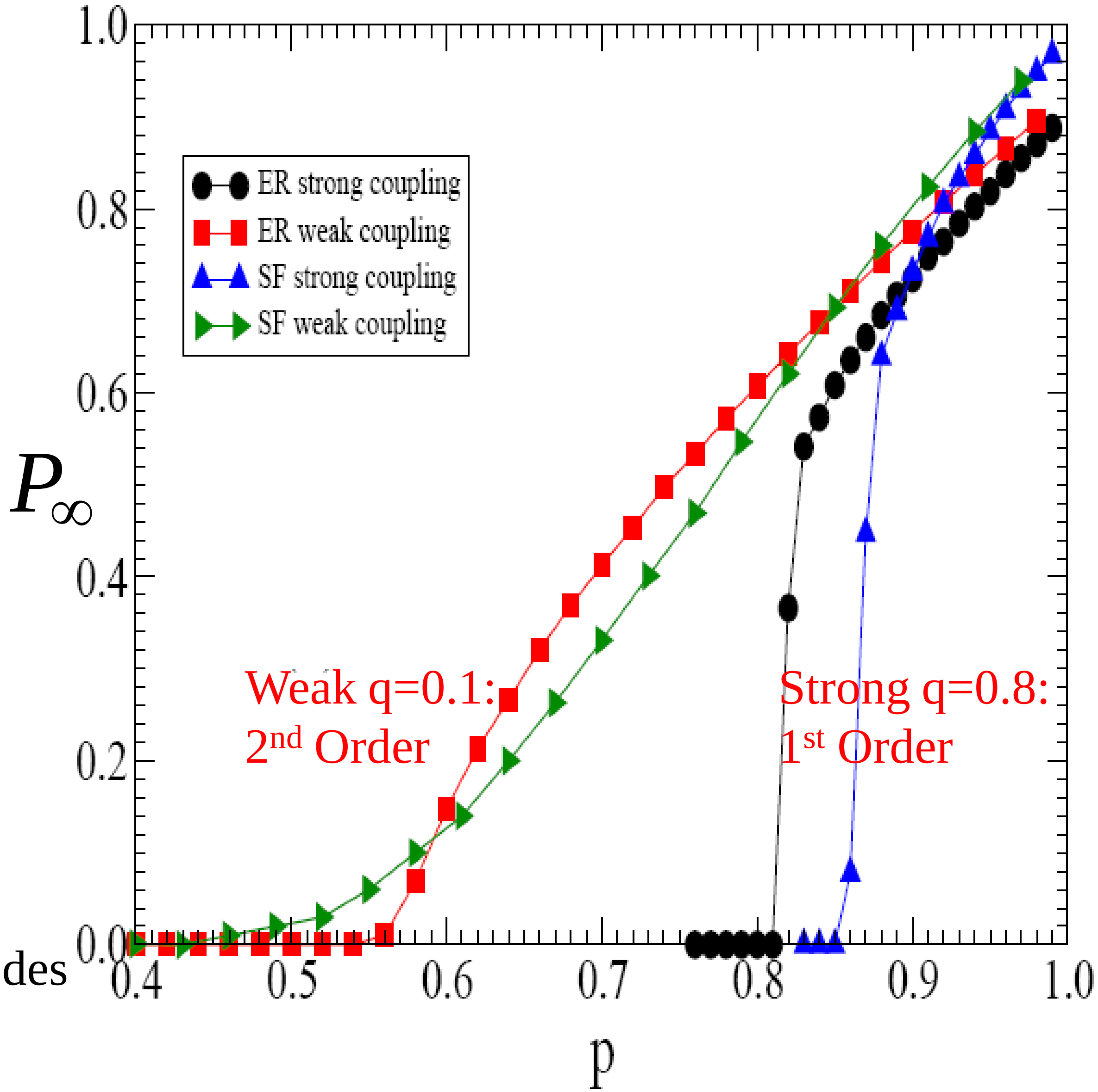
PDF of number of cascades n at criticality for ER of size N



GENERALIZATION: PARTIAL DEPENDENCE: Theory and Simulations

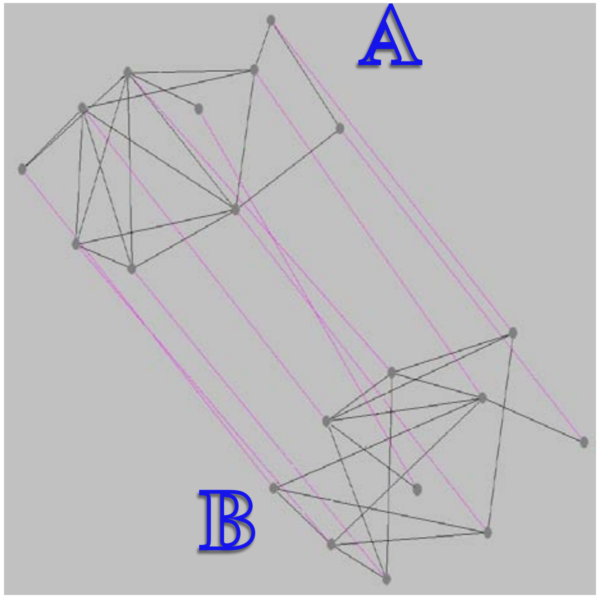


Parshani, Buldyrev, S.H.
PRL, **105**, 048701 (2010)

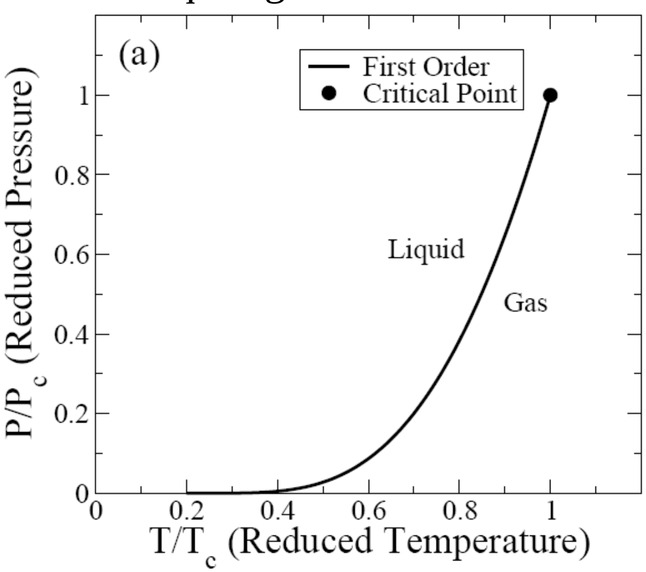


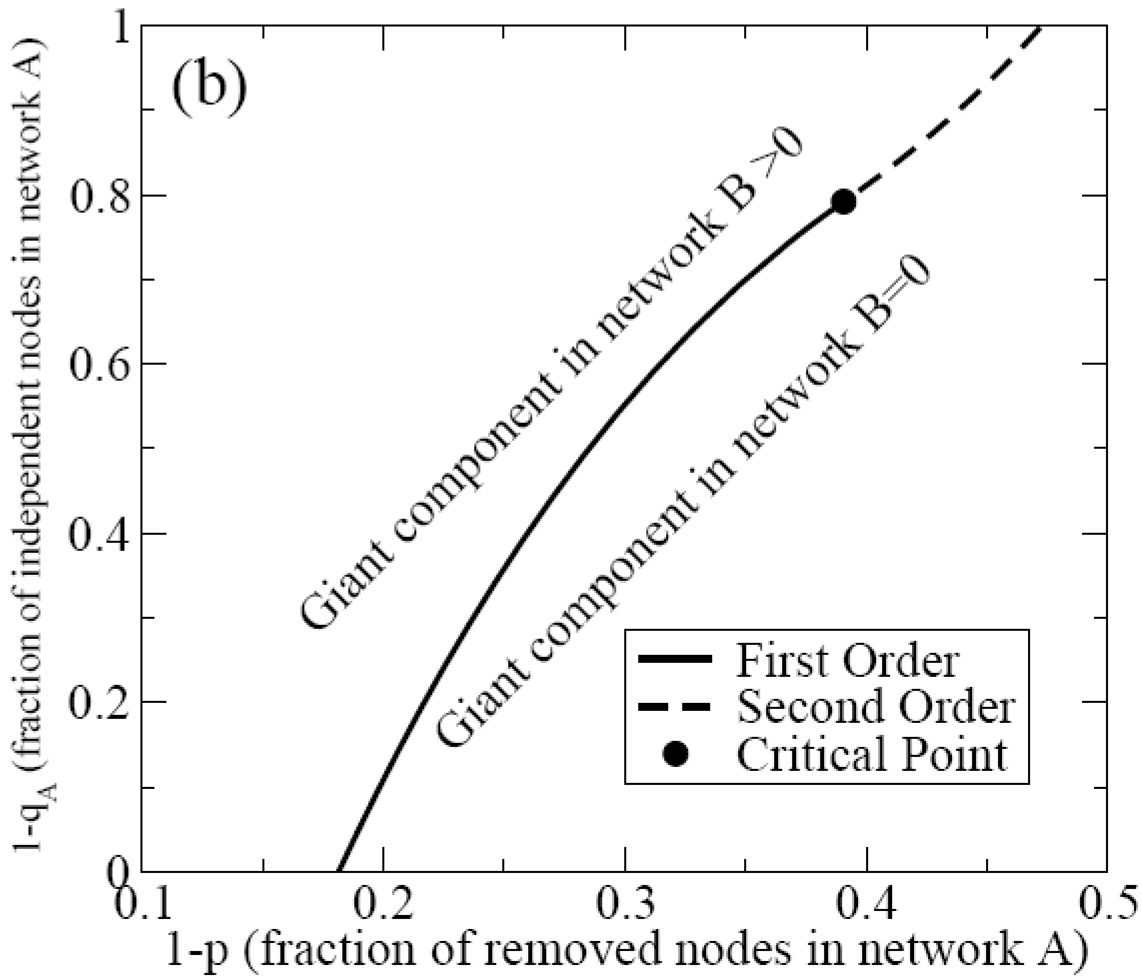
q-fraction of dependency nodes

PARTIAL DEPENDENCE: critical point



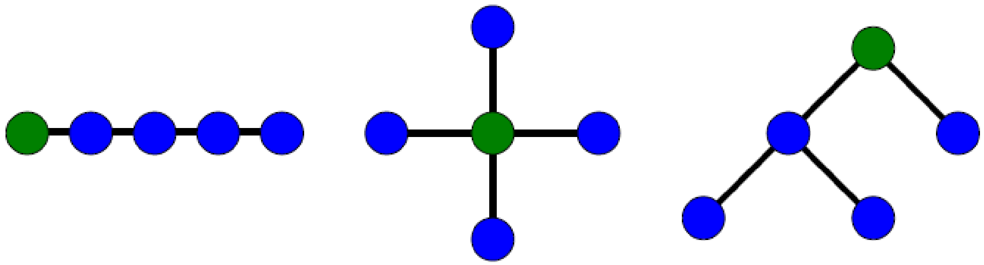
Analogous to critical point
in liquid-gas transition:





Network of Networks

m=5



For ER, $\langle k_i \rangle = k$, full coupling ,
ALL loopless topologies (chain, star, tree):

$$f_c = \exp\frac{f_c - 1}{mf_c}$$

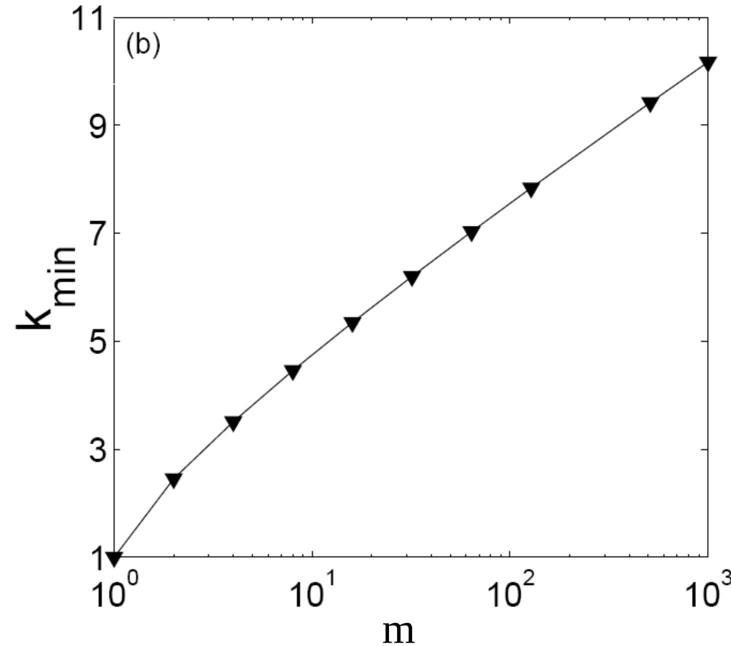
$$p_c = [mkf_c(1 - f_c)^{(m-1)}]^{-1}$$

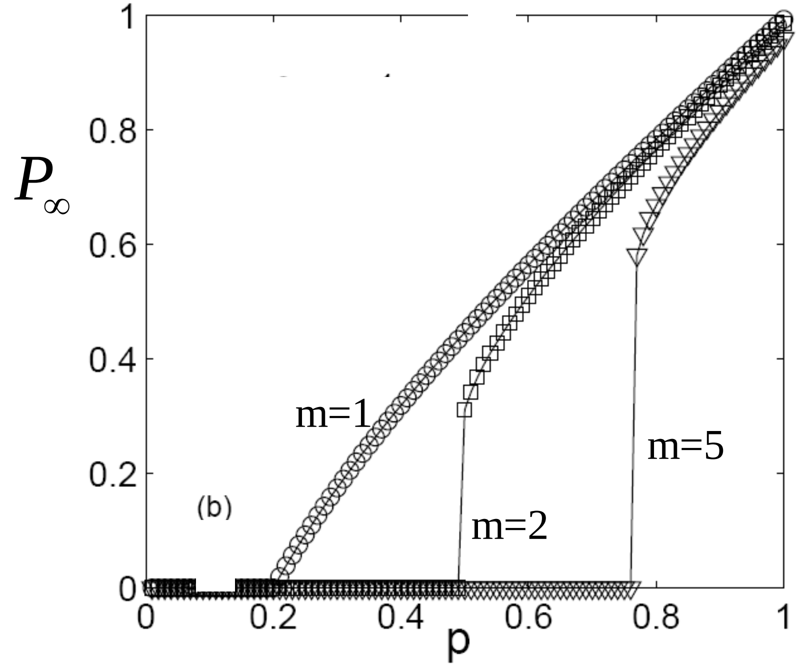
$$\boxed{P_\infty = p[1 - \exp(-kP_\infty)]^m}$$

m=1 known ER- 2nd order

$$p_c = 1/k$$

Vulnerability increases significantly with m





Jianxi Gao et al (arXiv:1010.5829)

Summary and Conclusions

- First statistical physics approach --mutual percolation-- for Interdependent Networks—cascading failures- 1st order transition

- Generalization to Partial Dependence:
 Strong coupling: first order phase transition; Weak: second order

- Generalization to Network of Networks: 50ys of classical percolation is a limiting case. E.g., only m=1 is 2nd order; m>1 are 1st order

$$P_\infty = p[1 - \exp(-kP_\infty)]^m \quad \text{(ER)}$$

- Extremely vulnerable: broader degree distribution-more robust in single networks becomes less robust in interacting networks

Rich problem: different types of networks and interconnections.

Buldyrev et al, NATURE (2010)
Parshani et al, PRL (2010);
Gao et al arXiv:1010.5829
Parshani et al, EPL (2010)
Parshani et al, PNAS (2011)
Huang et al, PRE (2011)