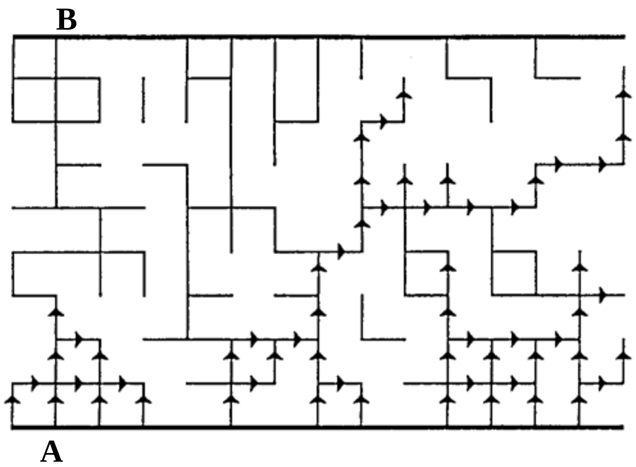
Directed Percolation

→Bond percolation on a square lattice

→Each bond has a direction towards $x>0$ or $y>0$

→Current can flow only in the arrow direction

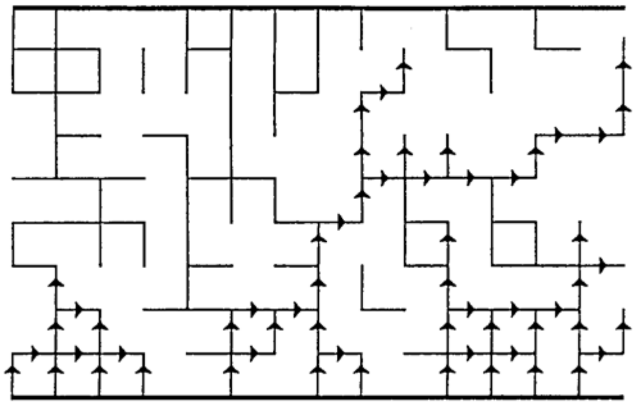


❖ Model for forest fire spreading under the influence of a wind

❖ Model for current in random diodes network

❖ Model for surface growth

Directed Percolation



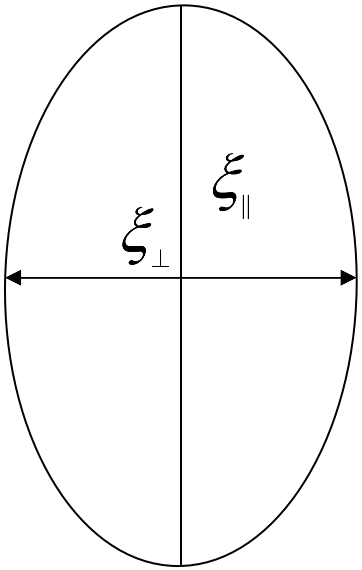
- There is a critical $p=p_c$ of directed bonds

- For $p<p_c$ no current flow from A to B
For $p>p_c$ current can flow!

- p_c is larger than p_c of isotropic percolation
For square lattice p_c =0.6447 (instead of p_c =0.5)
For triangular lattice p_c =0.479 (instead of p_c =0.35)

- The reason is that one needs to create a path without overhangs, d_{min}=1
(compared to d_{min}=1.13 in regular percolation)

Directed Percolation- Two correlation lengths



- The structure of directed percolation clusters is anisotropic
- Two correlation length:

$\xi_{\parallel}$ -in the percolation direction (x>0, y>0)

$\xi_{\perp}$ -perpendicular to percolation direction

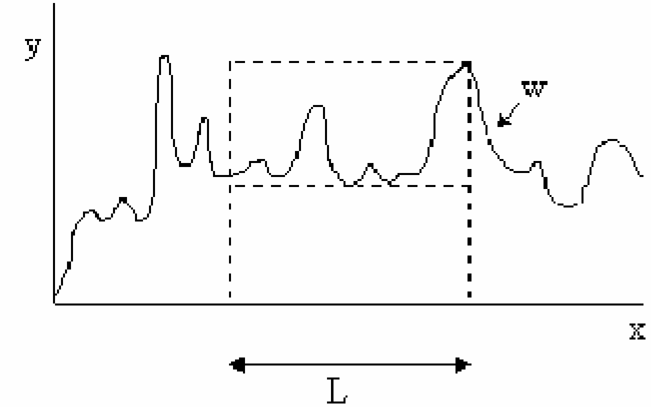
$$\xi_{\parallel} \sim |P - P_c|^{-\nu_{\parallel}} \qquad \xi_{\perp} \sim |P - P_c|^{-\nu_{\perp}} \qquad \nu_{\perp} < \nu_{\parallel}$$

- The clusters are therefore self affined
- For $d = 2$: $\nu_{\perp} \cong 1.097$ $\nu_{\parallel} \cong 1.733$
- A directed path will have a width $w \propto L^{\alpha}$

$$w \sim \xi_{\perp} \qquad L \sim \xi_{\parallel}$$

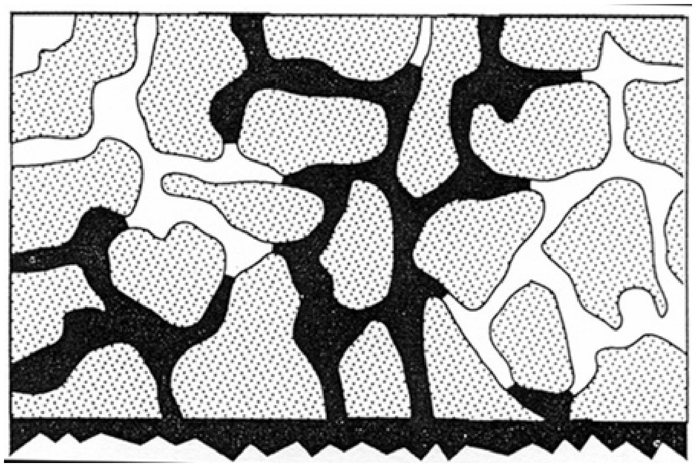
$$\xi_{\perp} \propto |P - P_c|^{-\nu_{\perp}} \propto \xi_{\parallel}^{\nu_{\perp}/\nu_{\parallel}} \sim \xi_{\parallel}^{0.67}$$

Thus $\underline{w \sim L^{0.63}}$



Invasion Percolation

❖ Flow of water into a porous media full of oil



❖ To extract oil from oil field usually one inserts water with high pressure in one hole and oil comes out from another hole

❖ Water and oil are incompressible fluids therefore when water invades into the rock oil comes out.

Invasion Percolation Model

❖ A lattice LxL full of oil

❖ Water invades from left bar

❖ Random numbers represent the resistance to invasion

❖ Water invades step by step in the smallest resistance sites

❖ This model is equivalent to PRIM and KRUSKAL algorithms for finding the "minimum spanning tree"

0.55	0.01	0.64	0.16	0.88
0.33	0.81	0.84	0.19	0.23
0.38	0.25	0.09	0.42	0.65
0.91	0.19	0.50	0.22	0.40
0.09	0.02	0.47	0.28	0.30

0.55	0.01	0.64	0.16	0.88
7	0.81	0.84	9	0.23
6	4	5	8	0.65
0.91	3	0.50	0.22	0.40
1	2	0.47	0.28	0.30

Invasion Percolation

- Since oil and water are incompressible liquids, regimes of oil surrounded by water can not be invaded any more
- Oil can be trapped in the porous media
- For d=2 d_f=1.82 < d_f=1.896 of regular percolation

  d_{min}=1.22 > d_{min}=1.13 of regular percolation
- These changes are due to the trapping
- For d=3 d_f =2.5 close to regular percolation

Anomalous Laws of Transport

❖ Due to fractal nature of percolation clusters physics laws such as diffusion, elasticity, conductivity become anomalous

❖ This is fractals do not have “translational symmetry” like crystals but “scale invariance” symmetry

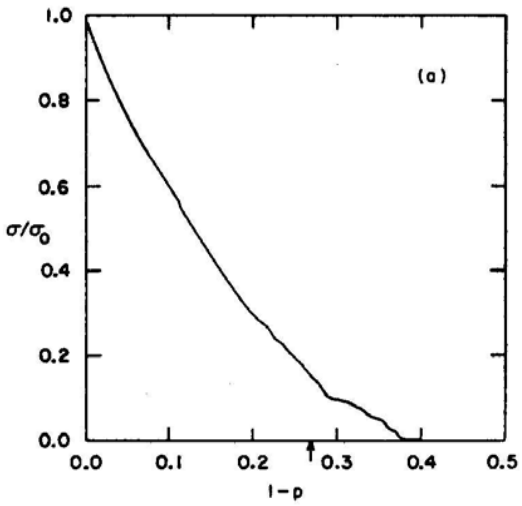
Diffusion and Conductivity:

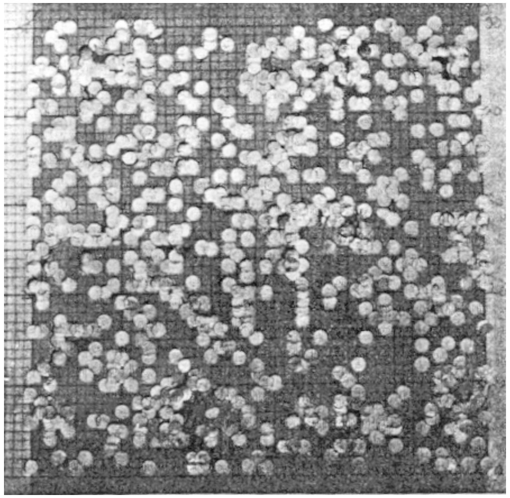
❖ Since percolation is a critical phenomena we expect physical laws will be a power of P-P_c near criticality

❖ The first experiment (Last and Thouless, 1971) measured conductivity σ is a diluted two dimensianal conductive material.

$$\sigma \sim (p - p_c)^{\mu}$$

$$P_c \approx 0.6 \qquad \mu \approx 1$$





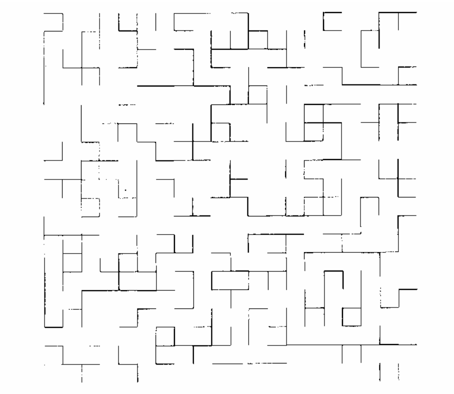
Conductive paper with concentration of 1-p = 0.268 holes

The conductivity as a function of concentration of holes

In a=3 the conductance of a mixture of AgCl (bad conductor) of concertration 1-p and AgI of concentration p

$$\sigma \sim (p - p_c)^{\mu}$$

$$P_c \approx 0.85 \qquad \mu \approx 2.3 \qquad (d = 3)$$



Simmulations:

- Generate LXL lattice resistance
- Each bond has a unit
- Remove randomly 1-p of the bond
- Calculate the total resistance $R \equiv {}^{1}\!/_{\sigma}$ as a function of *P*