

# Altruism: A natural strategy for enhancing survival

Alejandro F. Rozenfeld<sup>a,b</sup>, José Luis Gruver<sup>a</sup>,  
Ezequiel V. Albano<sup>b</sup>, Shlomo Havlin<sup>a,\*</sup>

<sup>a</sup>*Minerva Center and Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel*

<sup>b</sup>*INIFTA: Instituto de Investigaciones Fisicoquímicas Teóricas y Aplicadas, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, Sucursal 4, Casilla de Correo 16, (1900) La Plata, Argentina*

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## Abstract

We study the influence of altruistic behavior in a prey–predator model permitting the preys to commit suicide by confronting the predators instead of escaping. Surprising, altruistic behavior at microscopic (local) scale, leads to the emergence of new complex macroscopic (global) phenomena characterized by dramatic changes in the dynamic topology of the prey–predator spatiotemporal distribution, yielding spiral patterns. We show that such dynamics enhances the prey’s survivability.

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## 1. Introduction

Our work is motivated by the general question of how altruistic behavior could emerge as a natural strategy for improving survival. Along the history, altruism has been observed and discussed in quite diverse fields of science such as biology, ecology, sociology, economy and psychology [1–16].

Altruism can be defined as the capacity to perform costly acts that confer benefits on others [13], a concept that is often linked to cooperative behavior [10–13,15]. Within this context, there is a vast literature aimed to describe and understand altruism within the framework of evolutionary theories [8,9,13,15–17]. Furthermore, modelling altruism in the context of game theory (like the prisoner’s dilemma approach) has also attracted considerable theoretical attention [9,10].

Early mathematical approaches to achieving a quantitative description of competitive/cooperative behavior between individuals or species were formulated in terms of continuous models (i.e., differential equations) [2,3,18]. A well-known example is the Lotka–Volterra [19] model that has been a classic approach to studying the prey–predator dynamics. However, discreteness plays an important role in complex dynamic systems. Social, economic and biological organizations are inherently discrete (i.e., they are composed of discrete elements: individuals, companies, genes, animals, etc). In addition, the behavior of their members is realized in

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\*Corresponding author. Tel.: +972 3 531 8436; fax: +972 3 535 7678.

E-mail address: [havlin@ophir.ph.biu.ac.il](mailto:havlin@ophir.ph.biu.ac.il) (S. Havlin).

terms of discrete events. Consequently, a discrete mathematical representation of those complex organizations is in general expected to be more realistic. Indeed, it has been recently shown that life will always prevail in two-dimensional Lotka–Volterra-like systems as a result of discreteness, while the continuous approach predicts the contrary [20]. In the context of prey and predator systems, several mathematical models have been developed in the last decade including competitive and selfish behavior [21–26].

Although the various types of altruism widely found in nature, social sciences and economy may be quite complex, the use of simple approaches to altruistic and collaborative behavior is a very important first step for the development of more sophisticated models of general social behavior. Inspired by the classic Lotka–Volterra concepts, the aim of this work is to propose and study a discrete model that describes the dynamics of smart predators and altruistic preys that confront predators instead of escaping. Also, smart predators are able to locate and pursuit preys. As a counter intuitive consequence of the altruistic behavior, it is shown that the greater the fraction of altruistic preys is, the greater the final population of survival preys, and the lower is the population of predators.

## 2. Description of the model and simulation details

The model is formulated and simulated on a two dimensional square lattice of size  $L$ . A lattice site can be either empty or occupied by a single prey or predator. The model is studied by means of Monte Carlo simulations on the square lattice, measured in Lattice Units (LU), with periodic boundary conditions. During a Monte Carlo time step (mcs) all sites are updated once, on average. All the simulations start with a random initial distribution of the preys and predators over the lattice.

The system evolves according to consecutive cycles: (i) Interaction between the species and (ii) Altruistic confrontation and Smart escape-pursuit (AS) dynamics. Interactions are ruled as follows: (a) Preys give an offspring occupying an empty next neighbor site with probability BH (birth probability of preys) in case of absence of predators within their VH (visual range of preys). (b) Predators can eat a prey that exists in their MP (movement range of predators) with probability DH (death probability of preys). (c) Predators that have previously eaten a prey can give an offspring in the site previously occupied by the eaten prey, with probability BP (birth probability of predators). (d) Predators die with probability DP (death probability of predators). The AS rules are as follows: (a) A fraction  $f$  of altruistic preys moves into an empty site in the direction of maximal predator density while  $1 - f$  non-altruistic preys move into an empty site in the direction of minimal predator density. (b) Predators always move into an empty site in the direction of maximal prey density. (P) predators ((H) preys) can detect the presence of preys (predators) within their visual range VP (VH).

It is worth discussing on the type of altruism that is introduced in our prey–predator model. On the one hand, it is clear that the sacrifice of the preys can be considered as a kind of *strong altruism*, in the sense that it involves an absolute cost for the altruists [17,27,28]. Early models were interpreted as showing that strong altruism cannot occur in randomly formed groups [27,28]. However, very recently it has been demonstrated, by means of analytical calculations and numerical results, that altruism absent in random groups when they are formed, can emerge even after a single generation [17]. Of course, our model does not account for the origin and the early stages of evolution of the altruistic behavior. Instead, we consider that altruism is very well established within members of the prey’s society at the moment of the study. Also, as proposed by Pepper [29], one possible classification of the degree of altruism involves at least two large types: the *only-one* trait such that none of the altruist’s benefits come back to itself, in contrast to the *whole-group* where the benefit is divided among all members of the group including the altruistic individual. So, it is very interesting to remark that the altruism introduced in our prey–predator model may be classified as an “stochastic” combination of the above definitions. In fact, preys confronting predators have some chance to survive and receive part of the benefit by contributing to the overall fitness of their group, in contrast to those preys that are trapped by predators.

We have restricted ourselves to investigating the dependence of the system on the predator birth (BP) and death (DP) probabilities, respectively. The remaining parameters are kept constant, namely  $MP = VP = VH = 1$ , in order to allow for comparisons of the new results with previous studies [25,26]. It should also be remarked that by changing the range of the interactions (provided that they remain finite) we have only observed qualitative changes in the overall critical behavior of the system. In fact, irreversible phase

transitions from prey–predator coexistence to predator extinction (see below) remain of second order and in the universality class of directed percolation [25].

For additional details on the simulation method see Ref. [25].

### 3. Results and discussion

Fig. 1(a) shows the phase diagram corresponding to the prey–predator model studied here [25]. It is found that the system may evolve towards two different states or phases: (i) For  $DP \rightarrow 0$  (but  $DP > 0$ ) and  $BP \rightarrow 1$  (but  $BP < 1$ ), the final state of the system is a stationary regime with coexistence of preys and predators; (ii) for  $DP \rightarrow 0.5$  and  $BP \rightarrow 0$  (but  $BP > 0$ ), predators die out and surviving preys cover the whole sample. Of course, this phase is an absorbing state of the system where it becomes trapped irreversibly since the spontaneous birth of predators is not considered. At the boundary between these two phases a critical curve (solid line in Fig. 1(a)) can be determined. By means of extensive simulations we have already shown that the irreversible phase transition between species coexistence and the absorbing state belongs to the universality class of directed percolation [25,26]. This is quite natural since the absorbing state is unique and the transition is described by a scalar order parameter [30]. On the other hand, it is worth mentioning that, within the coexistence state, one can observe two different regimes. It is found that close to the critical curve the system reaches (after a transient period) a stationary density of species as shown in Fig. 1(b), which was obtained for  $DP = 0.25$  and  $BP = 0.40$  (recall that for this value of  $DP$  one has the critical point at  $BP_c = 0.351$ ). However, away from criticality ( $BP = 0.50$  in Fig. 1(c)) the system exhibits a self-sustained oscillatory with a well defined natural period [25,31]. Further detail on the behavior of the prey–predator model have already been published [25,26,31].

In order to study the influence of the altruistic behavior of the preys on the model we assume  $DP = 0.25$  through-all while  $BP$  is close to the critical curve. So, we have first evaluated the fraction of runs that the system achieves a prey–predator coexistence regime, as a function of the size of the lattice, for different values of  $f$  (see Fig. 2(a)). It is found that fluctuations in the populations may lead to the extinction of both species in small lattices, as observed in confined ecological systems. However, Fig. 2(a) suggests that all systems arrive at

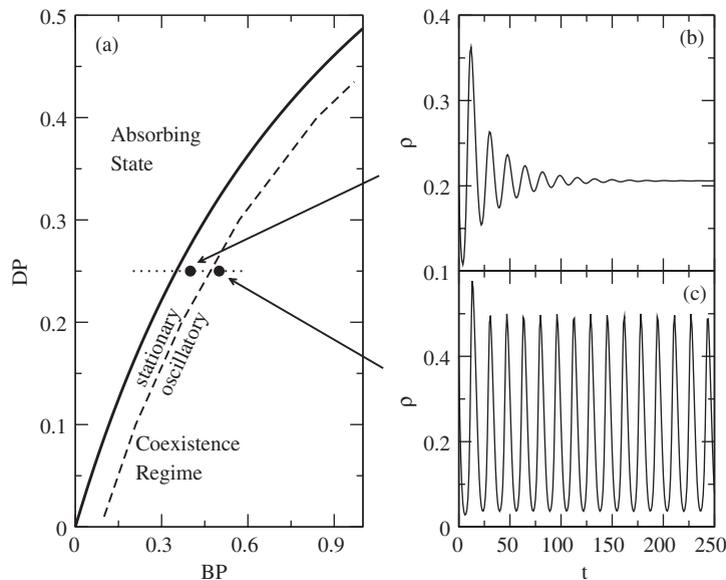


Fig. 1. Phase diagram of the prey–predator system as obtained by means of Monte Carlo simulations. (a) Plot of  $DP$  versus  $BP$  showing (i) the critical curve (solid line) for the irreversible phase transition between the prey–predator coexistence regime and the absorbing state where predator become extinct; (ii) the critical curve (dashed line) between the fixed point regime with stationary densities of species and the oscillatory behavior. (b) and (c) show plots of the prey density ( $\rho(t)$ ) versus  $t$  as obtained within the fixed point and oscillatory regime, respectively.

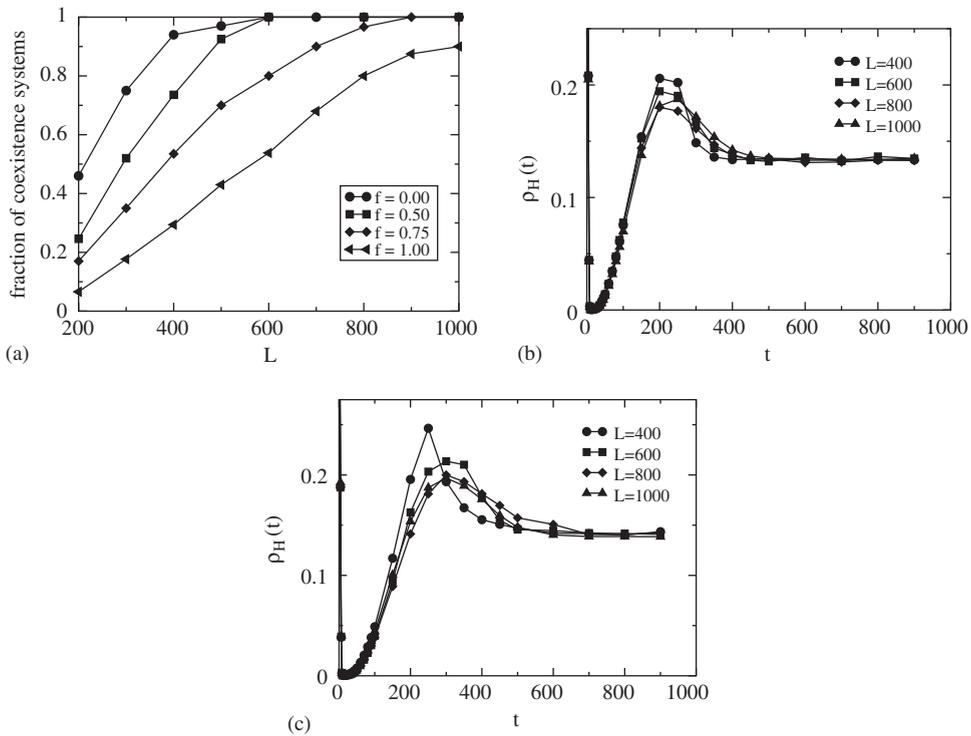


Fig. 2. (a) Plot of the fraction of systems that remain in prey–predator coexistence, as a function of the system size for different values of the fraction of altruistic preys ( $f$ ). Results are averaged over  $10^3$  different initial configurations. The plot suggests that for large enough lattices all systems will remain in coexistence irrespective of the value of  $f$ . (b–c) Plots of the temporal behavior of prey densities ( $\rho_H(t)$ ) for different sizes  $L$  of the sample. Results obtained for (b)  $f = 0.1$  and (c)  $f = 0.5$ , respectively. Averages are only taken over systems that achieve coexistence. Note that the stationary value of  $\rho_H$  is independent of  $L$ .

a coexistence regime for large enough lattices. Also, the stationary density of both species in coexistence systems is independent of the system size for all values of  $f$  (see Fig. 2(b–c) for the case of preys). Therefore, both finite and infinite coexistence systems will achieve the same stationary prey (predator) densities. This finding enable us to consider averages only in those finite systems that remain in coexistence and conclude about the stationary densities at very large systems.

The dependence of the stationary density of species on the fraction of altruistic preys has been studied using lattices of size  $L = 400$ , as shown in Fig. 3. Along the initial transient behavior both densities increase and spatiotemporal correlations start to develop. The inset of Fig. 3 shows the stationary densities as a function of  $f$ . From Fig. 3 follows that the prey density increases monotonically as the fraction of altruistic preys increases (see inset). In contrast, we find (not shown here for the sake of space) that the predator stationary density decreases monotonically. The result that altruistic behavior enhances the survival of preys, which is counter intuitive, constitutes one of the main conclusions of our study.

The fact that the density of preys increases as the fraction of altruistic preys grows can be understood in terms of the topology structure of the spatiotemporal patterns observed in the population space.

Fig. 4(a) and (b) shows typical snapshot configurations obtained for  $f = 1$  and of  $f = 0$ , respectively. In both cases, the individuals organize themselves in groups of preys closely followed by groups of predators. As seen from Fig. 4(a), due to the altruistic behavior, the topology is mainly governed by spirals in contrast to the turbulent ring-like structures observed when altruism is not present (Fig. 4(b)). Furthermore, the number of spirals increases as this fraction also increases. The spiral structure generated by the altruistic preys is the key to understanding the enhanced survival. Consider a moving group of preys followed by a group of predators. Altruistic preys placed at borders of the group would take the chance to move toward the predators. This strategic movement surrounding the predator group initiates the formation of a rotating pattern that will

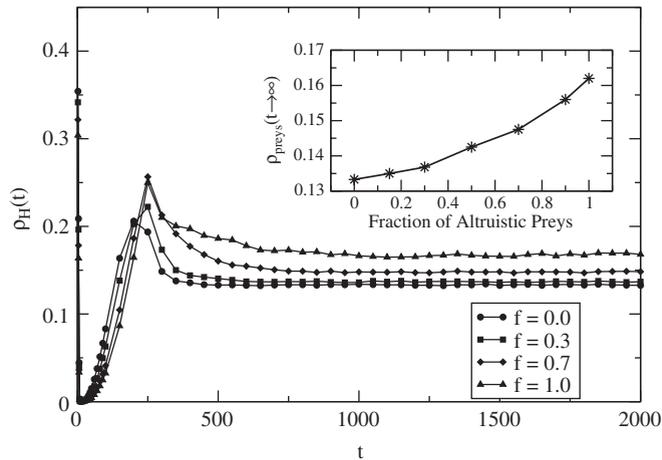


Fig. 3. Temporal series of the prey density ( $\rho_H(t)$ ) for different values of the fraction of altruistic preys  $f$ . The inset shows the dependence of averaged stationary prey density on the fraction of altruistic preys.

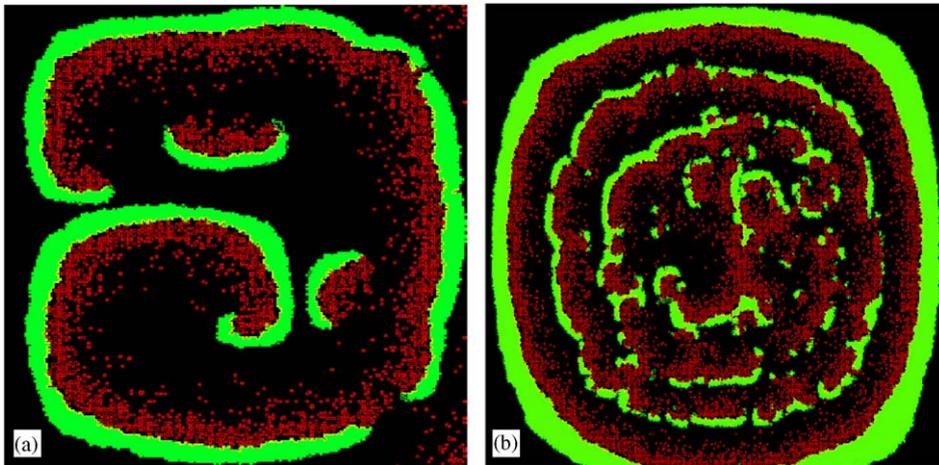


Fig. 4. (Colour online.) (a) Snapshot configurations of the prey–predator space when the whole group behaves in an altruistic manner. Note the spiral structure. (b) Typical spatial distribution generated when no altruistic preys are present in the system. Preys (predators) are shown in green (red).

give rise to a spiral (Fig. 4(a)). The spiral structure acts as a barrier, enabling preys to ‘yard’ predators. Consequently, more space will be free of predators, and suitable for prey offspring. In addition, the occurrence of this spiral patterns makes it possible to minimize the interface between preys and predators, and the probability that a group of preys may become surrounded by predators, and subsequently annihilated, is negligibly small. Accordingly a less favorable scenario for predator success emerges.

#### 4. Conclusions

We have studied a two-dimensional model of competitive population dynamics where the strongest form of altruism (i.e., offering the own life in order to save another’s life) was introduced at a microscopic level. It is shown that increasing the fraction of altruistic individuals the survivability of the population also increases. Moreover, it reaches its maximum when the whole population is altruist. Besides, we have also explained the observed phenomenon in term of the prey–predator topology space.

Regrettably it has to be admitted that we are unable to perform a direct comparison to biological facts, so our results may be considered as theoretical predictions. However, it should also be recognized that, for the moment, such kind of comparisons are scarce and in most cases would require a major experimental effort [12].

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