An Introduction to Interdependent Networks

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Abstract. Many real-world phenomena can be modelled using networks. Often, these networks interact with one another in non-trivial ways. Recently, a theory of interdependent networks has been developed which describes dependency between nodes across networks. Interdependent networks have a number of unique properties which are absent in single networks. In particular, systems of interdependent networks often undergo abrupt first-order percolation transitions induced by cascading failures. Here we present an overview of recent developments and significant findings regarding interdependent networks and networks of networks.

1 Background: From Single Networks to Networks of Networks

The field of network science emerged following the realization that in many complex systems, the way in which objects interact with each other could neither be reduced to their proximity (as in most traditional physical systems) nor was it random (as described in classical graph theory [1,2,3]). Rather, the topology of connections between objects in a wide range of real systems was shown to exhibit a broad spectrum of non-trivial structures: scale-free networks dominated by hubs [4,5], small-world networks which captured the familiar "six degrees of separation" idea [6,7] and countless other variations [8,9].

The late 1990s and early 2000s was an incredibly fruitful period in the study of complex networks and increasingly sophisticated models and measurements were developed. A strong catalyst for the growth of network science in this period was the rapid improvement of computers and new data from the world-wide web. The topological structure of networks was demonstrated to provide valuable insight into a wide range of real-world phenomena including percolation [10], epidemiology [11], marketing [12] and climate studies [13] amongst many others.

One of the most important properties of a network that was studied was its robustness following the failure of a subset of its nodes. Utilizing percolation theory, network robustness can be studied via the fraction occupied by its largest connected component P_∞ which is taken as a proxy for functionality of the network [14,15]. Consider, for example, a telephone network composed of

V.M. Mladenov and P.C. Ivanov (Eds.): NDES 2014, CCIS 438, pp. 189–202, 2014.

telephone lines and retransmitting stations. If $P_\infty \sim 1$ (the entire system), then information from one part of the network has a high probability of reaching any other part. If, however, $P_\infty \sim 0$, then information in one part cannot travel far and the network must be considered nonfunctional. Even if $P_\infty \sim 1$, some nodes may be detached from the largest connected component and those nodes are considered nonfunctional. We use the term *giant connected component* (GCC) to refer to P_∞ when it is of order 1. Percolation theory is concerned with determining $P_\infty(p)$ after a random fraction $1-p$ of nodes (or edges) are disabled in the network. Typically, $P_\infty(p)$ undergoes a second-order transition at a certain value p_c: for $p > p_c$, $P_\infty(p) > 0$ and it approaches zero as $p \to p_c$ but for $p < p_c$, $P_\infty(p) \equiv 0$. Thus there is a discontinuity in the derivative $P'_\infty(p)$ at p_c even though the function itself is continuous. It is in this sense that the phase transition is described as second-order [16,17]. It was shown, for example, that scale-free networks (SF)–which are extremely ubiquitous in nature–have $p_c = 0$ [10]. This is in marked contrast to Erdös-Rënyi (ER) networks ($p_c = 1/\langle k \rangle$) and 2D square lattices ($p_c = 0.5927$ [15]) and helps to explain the surprising robustness of many systems (e.g. the internet) with respect to random failures [18,10].

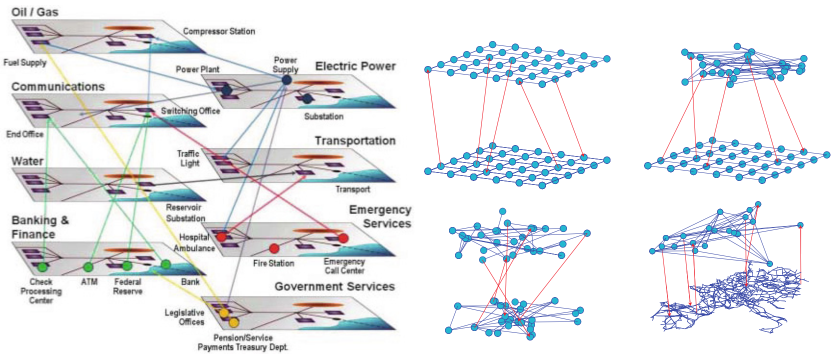
However, in reality, networks rarely appear in isolation. In epidemiology, diseases can spread within populations but can also transition to other populations, even to different species. In transportation networks, there are typically highway, bus, train and airplane networks covering the same areas but behaving differently [19]. Furthermore, the way in which one network affects another is not trivial and often specific nodes in one network interact with specific nodes in another network. This leads to the concept of interacting networks in which links exist between nodes within a single network as well as across networks. Just as ideal gases–which by definition are comprised of non-interacting particles–lack emergent critical phenomena such as phase transitions, we will see that the behavior of interacting networks has profound emergent properties which do not exist in single networks.

Since networks interact with one another selectively (and not generally all networks affecting all other networks), we can describe *networks of networks* with topologies between networks that are similar to the topology of nodes in a single network.

Multiplex networks are networks of networks in which the identity of the nodes is the same across different networks but the links are different [20,21,22]. Multiplex networks were first introduced to describe a person who participates in multiple social networks [23]. For instance, the networks of phone communication and email communication between individuals will have different topologies and different dynamics though the actors will be the same [24]. Also, each online social network shares the same individuals though the network topologies will be very different depending on the community which the social network represents.

One question that arises naturally in the discussion of networks of networks is, why describe this phenomenon as a "network of networks?" If we are describing a set of nodes and links then no matter how it is partitioned it is still a network.

Each description of interacting networks will answer this question differently but any attempt to describe a network of networks will be predicated on a claim that more is different—that by splitting the overall system into component networks, new phenomena can be uncovered and predicted. One way of describing the interaction between networks which yields qualitatively new phenomena is *interdependence*. This concept has been studied in the context of critical infrastucture and been formalized in several engineering models [25,26] (see Fig. 1a). However, as a theoretical property of interacting networks, interdependence was first introduced in a seminal study by Buldyrev et. al. in 2010 [27]. The remainder of this review will focus on the wealth of new phenomena which have been discovered using the ideas first described in that work.



(a) **(b)**

Fig. 1. An example of interdependent critical infrastructure systems and several modelled interdependent networks. (a) Schematic representation of interdependent critical infrastructure networks after [28]. (b) Illustration of interdependent networks composed of connectivity links (in blue, within the networks) and dependency links (in red, between the networks). Clockwise from upper-left: coupled lattices, a lattice coupled with a random regular (RR) network, two coupled RR networks and an RR network coupled to a real-world power grid. After [29]

2 Interdependence: Connectivity and Dependency Links

The fundamental property which characterizes interdependent networks is the existence of two qualitatively different kinds of links: *connectivity* links and *dependency* links [27,30,31] (see Fig. 1b). The connectivity links are the links which we are familiar with from single network theory and they connect nodes within the same network. They typically represent the ability of some quantity (information, electricity, traffic, disease etc) to flow from one node to another. From the perspective of percolation theory, if a node has multiple connectivity links leading to the GCC, it will only fail if all of those links cease to function. Dependency links, on the other hand, represent the idea that for a node to function, it requires support from another node which, in general, is in another network. In such a case, if the supporting node fails, the dependent node will also fail–even if it is still connected to the GCC in its network. If one network *depends on* and *supports* another network, we describe that pair of networks as interdependent. Interdependence is a common feature of critical infrastructure (see Fig. 1a) and many multiplex networks–often whatever causes a node to stop functioning in one layer will also disable it in other layers. Indeed, the percolation properties of interdependent networks describe the typical behavior in multiplex networks as well [27]. The properties of interdependence can affect a network's function in a variety of ways but here we focus on the response of a network of interdependent networks to the failure of a subset of its nodes using the tools of percolation theory [9]. We refer the reader to recent general reviews for other descriptions of interacting networks [23,32].

Percolation on a single network is an instantaneous process but on a system of interdependent networks, the removal of a random fraction $1-p$ of the nodes initiates a cascading failure in the following sense. Consider percolation on two interdependent networks A and B for which every node in A depends on exactly one node in B and vice versa. If we remove a fraction $1-p$ of the nodes in A, other nodes in A which were connected to the GCC via the removed nodes will also be disabled, leaving a new GCC of size $P_\infty(p) < p$. Since all of the nodes in B depend on nodes in A, a fraction $1-P_\infty(p)$ of the nodes in B will now be disabled via their dependency links. This will lead, in turn, to more nodes being cut off from the GCC in B and the new GCC in B will be smaller yet. This will lead to more damage in A due to the dependency links from B to A. This process of percolation and dependency damage accumulating iteratively continues until no more nodes are removed from iteration to iteration. This cascading failure is similar to the cascades described in flow and overload models on networks and the cascading failures in power grids which are linked to blackouts [33,34]. The cascade triggered by a single node removal has been called an "avalanche"[35] and the critical properties of this process have been studied extensively [35,36].

3 Interdependent Random Networks

This cascading failure was shown to lead to abrupt first-order transitions in systems of interdependent ER and SF networks that are qualitatively very different from the transitions in single networks (see Fig. 2b). Furthermore, p_c of a pair of ER networks was shown to increase from $1/\langle k\rangle$ to $2.4554/\langle k\rangle$. Surprisingly, it was found that scale-free networks, which are extremely robust to random failure on their own [18,10], become more vulnerable than equivalent ER networks when they are fully interdependent and for any $\lambda > 2$, $p_c > 0$. In general, a broader degree distribution leads to a higher p_c [27]. This is because the hubs in one network, which are the source of the stability of single scale-free networks, can be dependent on low degree nodes in the other network and are thus vulnerable to random damage via dependency links. These results were first demonstrated using the generating function formalism [37,27], though it has recently been shown that the same results can be obtained using the cavity method [38].

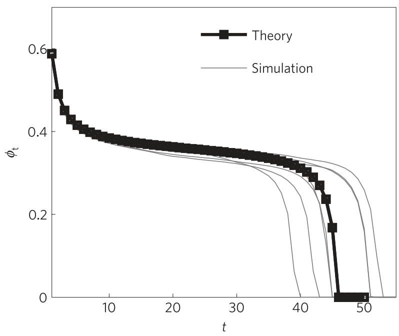
After the first results on interdependent networks were published in 2010 [27], the basic model described above was expanded to cover more diverse systems. One striking early result was that if less than an analytically calculable critical fraction q_c of the nodes in a system of two interdependent ER networks are interdependent, the phase transition reverts to the familiar second-order transition [30]. However, for scale-free networks, reducing the fraction of interdependent nodes leads to a hybrid transition, where a discontinuity in P_∞ is followed by a continuous decline to zero, as p decreases, (Fig. 2b)[39]. A similar transition was found when connectivity links between networks (which were first introduced in [40]) are combined with dependency links [41]. It has been shown that the same cascading failures emerge from systems with connectivity and dependency links within a single network [42,43,44].

In a series of articles, Gao et. al. extended the theory of pairs of interdependent networks to networks of interdependent networks (NoN) with general topologies [45,46,47,48]. Within this framework, analytic solutions for a number of key percolation quantities were presented including size of the GCC at each time-step t (see Fig. 2a), the size of the GCC at steady state (see Fig. 2b), p_c and other values.

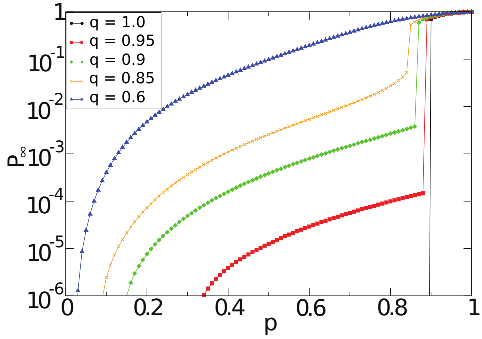
The NoN topologies which were solved analytically include: a tree-like NoN of ER, SF or random regular (RR) networks ($q = 1$), a loop-like NoN of ER, SF or RR networks ($q \leq 1$), a star-like NoN of ER networks ($q \leq 1$) and a RR NoN of ER, SF or RR networks ($q \leq 1$). For tree-like NoNs, it was found [45,47] that the number of networks in the NoN (n) affects the overall robustness but the specific topology of the NoN does not. In contrast, for a RR NoN the number of networks n does not affect the robustness but the degree of each network within the NoN (m) does[46,48]. Because the topology of the loop-like and RR NoNs allows for chains of dependency links going throughout the system, there exists a quantity q_{max} above which the system will collapse with the removal of a single node, even if each network is highly connected ($p = 1$).

In light of these results, we can now see that single network percolation is simply a limiting case of NoN percolation theory. Furthermore, this framework can be extended to predict the percolation properties of NoNs of arbitrary networks, provided the percolation profile of the individual networks are known, even only numerically [49]. A more detailed summary of these results was recently published in [31].

The assumption that each node can depend on only one node was relaxed in [50] and it was shown that even if a node has many redundant dependency links, the first-order transition described above can still take place. If dependency links are assigned randomly, a situation can arise in which a chain of dependency links can be arbitrarily long and thus a single failure can propagate through the entire system. To avoid this scenario, most models for interdependent networks assume uniqueness or "no feedback" which limits the length of chains of dependency links [46,48] For a pair of fully interdependent networks, this reduces to the requirement that every dependency link is bidirectional. Under partial de-



(a)



(b)

Fig. 2. Percolation of interdependent random networks. (a) The fraction of viable nodes at time t for a pair of partially interdependent ER networks. The gray lines represent individual realizations, the black squares are averages of all the realizations and the black line is calculated analytically. After [46]. (b) Percolation in a system of partially interdependent SF networks. We find three different kinds of phase transitions: first-order for $q = 1$ (black line), second-order for $q = 0.6$ and hybrid for $0.6 < q < 1$. This figure was generated for a system with $\lambda = 2.7$. After [39]. Cf. [41] for similar hybrid transitions.

pendency, this assumption is not necessary and the differences between systems with and without feedback have also been studied [48,29].

Though both the connectivity and dependency links were treated as random and uncorrelated in Refs. [27,30,45,46,47,48], the theory of interdependent networks has been expanded to more realistic cases. Assortativity of connectivity links was shown to decrease overall robustness [51]. Assortativity of dependency links was treated numerically [52] , analytically for the case of full degree-degree correlation [53] and analytically for the general case of degree-degree correlations with connectivity or dependency links using the cavity method [38]. Interestingly, if a fraction α of the highest degree nodes are made interdependent in each network, a three-phase system with a tricritical point emerges in the α–p plane [54]. If the system is a multiplex network, there may be overlapping links, i.e., two nodes which are linked in one layer may have a tendency to be linked in other layers [55,56,57]. In interdependent networks this phenomenon is referred to as intersimilarity [52,58]. Clustering, which has a negligible effect on the robustness of single networks [59], was shown to substantially reduce the robustness of interdependent networks [60,39].

4 Spatially Embedded Interdependent Networks

One of the most compelling motivations for developing a theory of interdependent networks is that many critical infrastructure networks depend on one another to function [25,26]. Essentially all critical infrastructure networks depend on electricity to function, which is why threats like electromagnetic pulses are taken so seriously (see Fig. 1a, Ref. [28]). The power grid itself, though, requires synchronization and control which it can only receive when the communication network is operational. One of the largest blackouts in recent history, the 2003 Italy blackout, was determined to have been caused by a cascading failure between electrical and communications networks [61].

In contrast to random networks, all infrastructure networks are embedded in space [19]. The nodes (e.g., power stations, communication lines, retransmitters etc.) occupy specific positions in a 2D plane and the fact that the cost of links increases with their length leads to a topology that is markedly different from random networks [62]. Thus infrastructure networks will tend to be approximately planar and the distribution of geographic link distances will be exponential with a characteristic length [63]. From universality principles, all such networks are expected to have the same general percolation behavior as standard 2D lattices [14,63]. As such, the first descriptions of spatially embedded interdependent networks were modelled with square lattices [64,29,65,66,49] and the results have been verified on synthetic and real-world power grids [29,65].

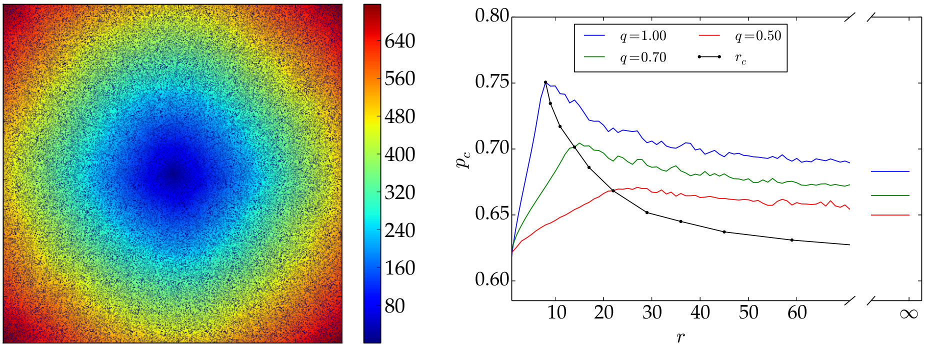
Analytic descriptions of percolation phenomena require that the network be "locally tree-like" and in the limit of large systems, this assumption is very accurate for random networks of arbitrary degree distribution [37]. However, lattices and other spatially embedded networks are not even remotely tree-like and analytic results on percolation properties are almost impossible to obtain [14,15]. Therefore most of the results on spatially embedded networks are based on numerical simulations.

One of the few major analytic results for spatially embedded systems is that for interdependent lattices, if there is no restriction on the length of the dependency links then any fraction of dependency leads to a first-order transition ($q_c = 0$). In [29], it was shown that the critical fraction q_c for which the system transitions from the first-order regime to the second order regime must fulfill:

$$1 = p_c^{\star} q_c P'_{\infty}(p_c) \tag{1}$$

in which $p_c^{\star}$ is the percolation threshold in the system of interdependent lattices, p_c is the percolation threshold in a single lattice and $P'_{\infty}(p)$ is the derivative of $P_{\infty}(p)$ for a single lattice. Since as $p \to p_c$, $P_{\infty}(p) = A(x - p_c)^{\beta}$ and for 2D lattices $\beta = 5/36$ [67], $P'_{\infty}(p)$ diverges as $p \to p_c$ and the only way to fulfill Eq. 1 is if $q_c = 0$. From universality arguments, all spatially embedded networks in $d < 6$ have $\beta < 1$ [14,15,63] and thus all systems composed of interdependent spatially embedded networks (in $d < 6$) with random dependency links will have $q_c = 0$. In Fig. 1b, all of the configurations shown except the RR-RR system have $q_c = 0$.

If the dependency links are of limited length, the percolation behavior is surprisingly complex and a new spreading failure emerges. Li et. al. [64] introduced the parameter r, called the "dependency length," to describe the fact that in most systems of interest the dependency links, too, will likely be costly to create and, like the connectivity links, will tend to be shorter than a certain characteristic length. In this model, dependency links between networks are selected at random but are always of length less than r (in lattice units). If $r = 0$, the system of interdependent lattices behaves identically to a single lattice. If $r = \infty$, the dependency links are unconstrained and purely random as in [29]. Li et. al. found that as long as r is below a critical length $r_c \approx 8$ the transition is second-order but for $r > r_c$ the transition is first order [64] (See Fig. 3b). The first-order transition for spatially embedded interdependent networks is unique in that it is characterized by a spreading process. Once damage of a certain size emerges at a given place on the lattice, it will begin to propagate outwards and destroy the entire system (See Fig. 3a).



(a) (b)

Fig. 3. Percolation of spatially embedded networks. (a) A snapshot of one lattice in a pair of interdependent lattices with nodes colored according to the time-step in which the node failed. The regularity of the color-change reflects the constant speed of the spreading failure in space (Generated for $q = 1, r = 11, L = 2900$). (b) The effect of r and q on p_c. As r increases, p_c increases until r reaches r_c. At that point the transition becomes first-order and p_c starts decreasing until it reaches its asymptotic value at $r = \infty$. Both after [66].

If the dependency is reduced from $q = 1$ to lower values, it is found that r_c increases and diverges at $q = 0$, consistent with the result from [29] that $q_c = 0$ for $r = \infty$ [66] (See Fig. 3b). Recently, the framework developed in [45,46,47,48] was extended to general networks of spatially embedded networks. Similar to the case of networks of random networks, the robustness of tree-like spatially embedded NoNs are affected by n but not by the topology while RR NoNs are affected by m but not by n [49].

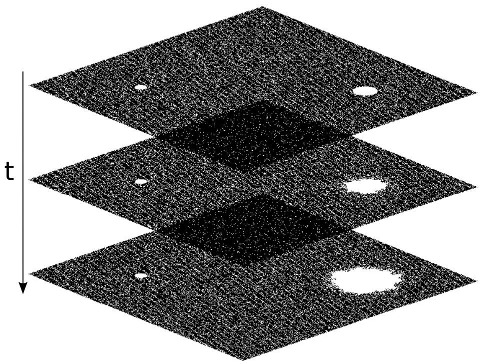
5 Attack and Defense of Interdependent Networks

Due to their startling vulnerabilities with respect to random failures, it is of particular interest to understand how non-random attacks affect interdependent networks and how to improve the robustness of interdependent networks through topological changes. Huang et. al. [68] studied tunable degree-targeted attacks on interdependent networks. They found that even attacks which only affected low-degree nodes caused severe damage because high-degree nodes in one network can depend on low-degree nodes in another network. This framework was later expanded to general networks of networks [69].

Since high degree nodes in one network which depend on low degree nodes in another network can lead to extreme vulnerability, there have been several attempts mitigate this vulnerability by making small modifications to the inter-network topology. Schneider et. al. [70] demonstrated that selecting autonomous nodes by degree or betweenness can greatly reduce the chances of a catastrophic cascading failure. Valdez et. al. have also obtained promising results by selecting a small fraction of high-degree nodes and making them autonomous [71]. These mitigation strategies are methodologically related to the intersimilarity/overlap studies discussed above.

As we have seen, cascading failures are dynamic processes and the overall cascade lifetime can indeed be very long [36,66]. The slowness of the process opens the door for "healing" methods allowing the dynamic recovery of failed nodes in the midst of the cascade. Recently, a possible healing mechanism along these lines has been proposed and analyzed [72].

When considering infrastructure or other spatially embedded networks, not only is the network embedded in space but failures are also expected to be geographically localized. For instance, natural disasters can disable nodes across all networks in a given area while EMP or biological attacks can disable the power



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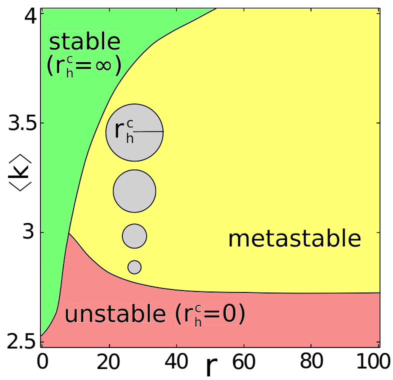

(b)

Fig. 4. Geographically localized attacks on interdependent networks. (a) The hole on the left is below r_h^c and stays in place while the hole on the right is larger than r_h^c and propagates through the system. (b) The phase space of localized attacks on interdependent networks. The increasing gray circles represent the dependence of r_h^c on $\langle k \rangle$. Both after [65].

grid or social network only in a given area. Geographically localized attacks of this sort have received attention in the context of single network percolation[73] and flow-based cascading failures [74]. However, the existence of dependency between networks leads to surprising new effects. Recently, Berezin et. al. [65] have shown that spatially embedded networks with dependencies can exist in three phases: stable, unstable and metastable (See Fig. 4b). In the metastable phase, the system is robust with respect to random attacks–even if finite fractions of the system are removed. However, if all of the nodes within a critical radius r_h^c fail, it causes a cascading failure which spreads throughout the system and destroys it (See Fig. 4a). Significantly, the value of r_h^c does not scale with system size and thus, in the limit of large systems, it constitutes a zero-fraction of the total system.

6 Applications of Networks of Networks

Many of the fields for which networks were seen as relevant models have been re-evaluated in light of the realization that interacting networks behave differently than single networks. Epidemics on interdependent and interconnected networks have received considerable attention [24,75,76,77,78]. Economic networks composed of individuals, firms and banks all interact with one another and are susceptible to large scale cascading failures [79,80] Interacting networks have also been found in physiological systems [81], ecology [82] and climate studies [83]. Multilevel transportation networks have also been studied from the perspective of interacting networks [84].

The breadth of applications of networks of networks is too great to address here and we refer the reader to recent reviews for more thorough treatment of applications [23,32].

Acknowledgments. We acknowledge the LINC (No. 289447) and MULTIPLEX (No. 317532) EU projects, the Deutsche Forschungsgemeinschaft (DFG), the Israel Science Foundation, ONR and DTRA for financial support.

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