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CAN STATISTICAL PHYSICS CONTRIBUTE TO THE SCIENCE OF ECONOMICS?

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Abstract

In recent years, a breakthrough in statistical physics has occurred. Simply put, statistical physicists have determined that physical systems which consist of a large number of interacting particles obey universal laws that are independent of the microscopic details. This progress was mainly due to the development of scaling theory. Since economic systems also consist of a large number of interacting units, it is plausible that scaling theory can be applied to economics. To test this possibility we study the dynamics of firm size. This may help to build a more complete characterization of the nature and processes behind firm growth.

To date, the study of firm dynamics has primarily focused on whether small firms on average have higher growth rates than large firms. To a lesser extent, attention has been placed on the relationship between firm size and variation in growth rate. Our research goes beyond these questions by looking at the relationship between numerous firm characteristics and the entire distribution of growth rates. Thus, it may provide a better understanding of the mechanisms behind firm dynamics. In contrast to previous studies, this research analyzes data over many time scales, instead of just a single time interval.

From a scientific standpoint, this work could be useful because it will affect the formulation of firm modeling — one of the basic building blocks of all economic analysis. In addition, this work will have practical applications. For example,

there are Federal policies that are designed to encourage small businesses. While such policies might be justified on grounds other than their contribution to growth, any systematic difference in the growth rates of small and large firms might be relevant for evaluating such policies. Also, there has traditionally been a concern that an excessive amount of economic activity might become concentrated in a small number of firms. A more detailed understanding of the firm growth process will provide evidence for whether such concerns have any scientific foundation.

1. INTRODUCTION

The objective of our recent research has been to use methods from statistical physics to provide a more complete description of the dynamics of firm growth. Economists have already studied the static distribution of firms[1–5] and the relationship between firm size on the one hand, and the mean and standard deviations of growth rates on the other.[2,6–8] A key exercise in statistical physics is to examine the distributions of observable features of complex dynamic systems at different scales. In studying turbulent fluids, for example, statistical physicists examine the evolution of the velocity distribution over time. In studying the dynamics of firms, two natural scales to study are size and time. From the standpoint of statistical physics, the relationship between size on the one hand, and the mean and standard deviations of the growth rates on the other is not a complete representation of firm dynamics. Rather, a complete description would include how the entire shape of the distribution varies with size. Similarly, every existing study of firm growth has examined growth rates over a single time scale. A complete description of the dynamics would include how the entire distribution of growth rates evolves over different time scales.

The theory of the firm is fundamental to economic analysis; yet, the classical model of the firm, in which the firm is viewed simply as a production function, is inadequate for many applications in economics. There is an older literature about alternative theories of the firm,[9–11] but the classical model has survived as the dominant one.[12] Recently, however, it has been recognized that more sophisticated models are needed for many applications, most notably in the areas of corporate finance, labor economics, law and economics, industrial organization, and management strategy. As a result, there has been a recent revival in research on the nature of the firm.[13–18] Much of this work has been theoretical. The objective of this research is to provide a set of empirical findings with which future theories of the firm must be consistent. There is a tradition in economics of reconciling theories of the firm with statistical analysis of firm size and growth. For example, both Lucas'[19] seminal paper on the distribution of firm size and Nelson and Winter's[15] work on firm and economic growth made use of Gibrat's Law.

Our results should have potential applications in many areas of economics. For example, ever since the seminal work of Solow,[20] it has been generally accepted that technical change (as opposed to capital accumulation) is the primary source of economic growth. There are a number of hypotheses about the relationship between firm size and efficiency in doing research and development (R&D).[21,22] A better understanding of the relationship between size and firm growth can provide insights into which (if any) of these hypotheses is correct. Another application is the growing literature on the role of internal capital markets in allocating funds for investment.[23–26] One view holds that internally generated funds are irrelevant for investment decisions because funds for profitable projects can be raised on the capital markets. The alternative view is that there are significant costs to external forms of finance so that the availability of internally generated funds does affect investment decisions. However, there are competing hypotheses about the implications of the importance of the availability of internally generated funds. Under the free cash flow hypothesis,[27] the availability of funds gives managers the freedom to pursue their own objectives at the expense of shareholders. The alternative is that the availability of funds makes it possible to finance profitable investments. An examination of the entire distribution of growth rates as a function of firm characteristics (including but not limited to size) can shed light on the relative importance of each of these hypotheses.

Before discussing the specifics of this research, we should note that this work represents a collaboration between scholars from two different disciplines: statistical physics and economics. The model of statistical physicists exploring topics in economics in daily collaboration with economists has great potential. The methodologies of economics and statistical physics have evolved in different ways and viewing economics through the lens of statistical physics, we can provide a fresh perspective. Currently, there are several preliminary results which, if confirmed by more extensive work, could be of significance.[5,28,29]

2. SCALING THEORY

Recently, statistical physicists have formulated scale invariance theory, also known as scaling theory. This has led to a breakthrough in our understanding of complex physical and chemical systems that consist of a large number of interacting particles.[30–33]

The main concept of scaling theory is that the properties of a many-particle system under certain conditions will become scale invariant, i.e., the behavior of a system at short distances or times is similar to the behavior of a system for long distances or times. Thus a unique scaling function can describe the whole range of phenomena. This scaling function does not depend on most of the microscopic details of the system, such as inter-molecular interactions or the structure of the molecules, but only on a few fundamental properties. In statistical physics, this phenomena is called universality. Thus, knowledge of the properties of a model system at short times or short length scales can be used to predict the behavior of a real system at large times and large length scales.

Since economic systems consist of a large number of interacting units, it is plausible that they might be amenable to scaling analysis. In fact, a recent study determined that the fluctuations in the S&P 500 index exhibited scaling behavior.[29] Using 1.5 million records at 1 minute intervals over 6 years of trading, Mantegna and Stanley determined that fluctuations on a one minute time interval were reflected in 10-minute, 100-minute, and 1000-minute intervals.[29] The distribution of index returns fits a Lévy (similar to a Pareto) distribution with an exponential drop off in the tails, as shown in Fig. 1. These scaling properties mean that viewing stock market returns on one minute intervals provides insight on 1000-minute intervals. Unlike other models of the stock market, e.g., ARCH/GARCH models,[34,35] Mantegna and Stanley demonstrated that even the crash of October 1987 — an event that occurs only once every several million minutes — can be characterized by the perturbations that obey scaling theory.

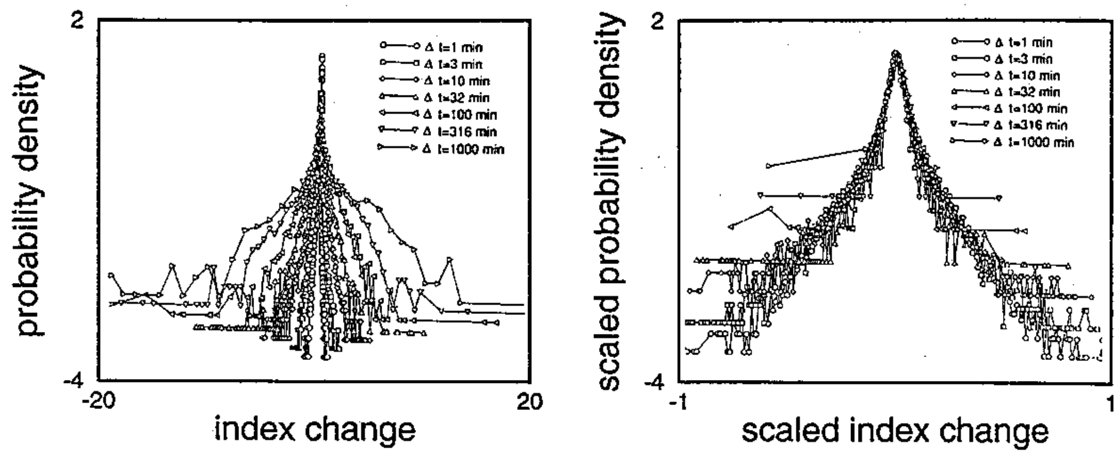

Fig. 1 (a) Probability distribution of the S&P 500 index variations observed at time intervals Δt which range from 1 to 1000 minutes. By increasing Δt, spreading of the probability distribution is observed. (b) By scaling the plot in (a), all the data collapses on the $\Delta t = 1$ min. distribution. The points outside the average behavior define the noise level of that specific distribution.

3. SIZE DISTRIBUTION OF FIRMS

3.1 Review of Recent Results

Gibrat[1] suggested a simple stochastic dynamic model of firm growth that predicts the distribution of firm size to be log-normal. Later research by Simon and Bonini[3] suggested that the distribution of the largest firms is a power law, resulting in an upper tail of the overall distribution that is too thick relative to the log-normal. Their result has to be viewed under the restrictions of the time. For example, for a comprehensive data set, they used the Fortune 500, which is only a list of the largest firms. Using both a more complete database, *Compustat* (which includes all publicly-traded US manufacturing firms), and techniques only recently developed in statistical physics, we confirmed that the log-normal fits the data reasonably well. However, in contrast to previous findings, we found that the upper tail of the distribution is too thin relative to the log-normal, not too fat.[5] In all likelihood, we could not have made this discovery without the use of "Zipf plots" — a technique of statistical physics used to highlight the upper tail of a distribution.

In Fig. 2(a), we show the histogram of the data fit by a log-normal curve. From this graph, it is not possible to discern any systematic deviations in the upper tail. However, Fig. 2(b) utilizes the "Zipf plot" and the deviation becomes remarkably pronounced.

3.2 Open Questions

Our results suggest that the sales of very large firms grow by an entirely different mechanism than the sales of smaller firms. Much work remains to be done to test the above ideas. A few of the questions that we seek to answer are:

- Does the distribution of other characteristics, such as the number of employees, exhibit a similar shape?

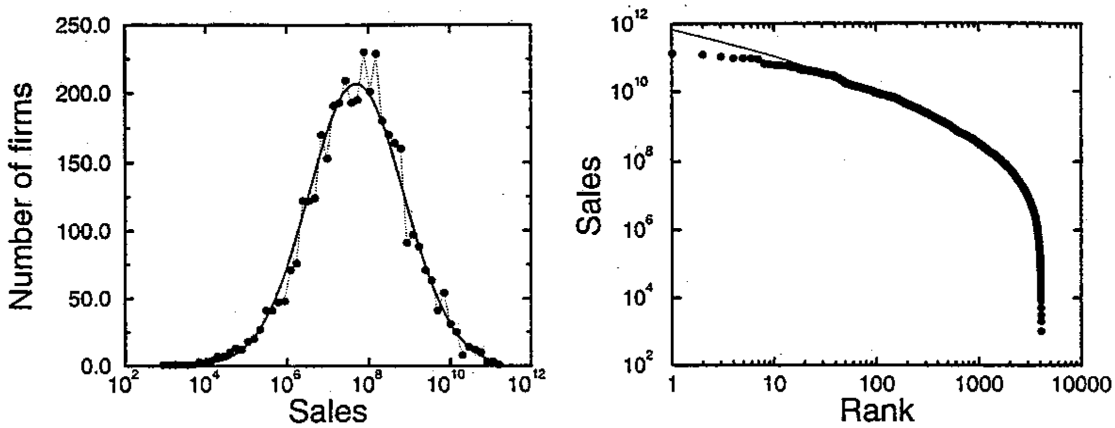

Fig. 2 (a) Histogram showing the number of firms as a function of 1993 sales. The data are for the 4071 *Compustat* firms with standard industrial classification codes 2000-3999. The value of the sales are binned in powers of $2^{1/2}$. The solid curve is a log-normal fit to the data using the mean of the log of sales and the standard deviation of the log of sales as fitting parameters. From the histogram, the deviations in the upper tail are not noticeable. (b) The data are a Zipf plot (double logarithmic plot of sales vs. rank) for the same sample as in (a). The solid line is a predicted Zipf plot from the log-normal fit shown in (a). From the Zipf plot, the deviations in the upper tail are easily noticed.

- Are the distributions of firms outside the manufacturing sector similar to the distribution in Fig. 2?
- How does the distribution in Fig. 2 appear for different countries? Do the distributions for different economies obey the scaling theory?

4. GROWTH RATE DISTRIBUTION

4.1 Review of Recent Results

In statistical physics, it is common to explore the underlying mechanisms of a complex dynamic process by studying the time evolution of distribution functions. Using the *Compustat* database over the time period 1975–1991, our results suggest:

(i) The distribution of annual logarithmic growth rates for firms with approximately the same size displays an exponential form.
(ii) The fluctuations in the growth rates — measured by the width (or standard deviation) of this distribution — decrease exponentially with the logarithm of firm size.

We made plots on a double logarithmic scale of the probability density function of the real growth rates in sales from 1990 to 1991. Independent of the initial sales, the curves display a simple "tent-shaped" form. Hence the distribution is not Gaussian — as expected from the Gibrat hypothesis[1] — but rather is exponential, with the probability density p obeying the following relation:

$$p(r_1|s_0) = \frac{1}{\sqrt{2}\sigma_1(s_0)} \exp\left(-\frac{\sqrt{2}|r_1 - \bar{r}_1(s_0)|}{\sigma_1(s_0)}\right), \tag{1}$$

where s_0 is the log of the initial year sales and r_1 is the logarithm of the growth rate, defined as the ratio of next years' sales over this year's (the initial year's) sales. We calculated the average growth rate $\bar{r}_1(s_0)$ and the standard deviation $\sigma_1(s_0)$ by fitting the data set to Eq. (1).

We also tested the robustness of the exponential function of Eq. (1) for other one-year intervals. We find that the data for *each* annual interval from 1975–1991 fit well into Eq. (1), with only small variations in the parameters $\bar{r}_1(s_0)$ and $\sigma_1(s_0)$. To improve the statistics, we therefore calculated a new histogram by averaging all the data from the 16 annual intervals in the database. The data, converting sales to 1987 constant dollars and combining, now scatter much less and the shape is well described by Eq. (1).

In any distribution, it is crucial to study its shape, its mean and its standard deviations. One notices that the larger firms grow less rapidly than the smaller ones. We quantitatively analyze the relation of the mean growth rate over one-year, $\bar{r}_1(s_0)$, and initial sales, s_0. The data suggests that while there is a slight trend toward slower growth in large firms, the mean growth rate is relatively constant for relatively large firms.

We also study the dependence on s_0 of $\sigma_1(s_0)$ which decreases with increasing s_0. We quantitatively analyze this feature by plotting $\sigma_1(s_0)$ vs. s_0, and we find that the standard deviation is well approximated over more than 7 orders of magnitude (from sales of less than 10^4 dollars up to sales of more than 10^{11} dollars) by the law:

$$\sigma_1(s_0) = a \exp(-\beta s_0) , \tag{2}$$

where $a \simeq 6.66$ and $\beta = 0.15 \pm 0.01$. In physical systems, it is not unusual to find such simple relationships holding over a wide range, but such a relation for economic data is striking and merits further analysis.

In order to further check the robustness of our findings, we performed a parallel analysis for the number of employees, which is also given in the *Compustat* database for each successive year. We find that the analogs of $p(r_1|s_0)$ and $\sigma_1(s_0)$ behave in the same fashion. For the standard deviation of the number of employees, we find that the data are linear over roughly 5 orders of magnitude, from firms with only 10 employees to firms with almost 10^6 employees. The slope $\beta = 0.16 \pm 0.01$ is the same, within error bars, as found for the sales.

In statistical physics, scaling phenomena of the sort that we have uncovered in the sales and employee distribution functions are sometimes represented graphically by plotting a suitably scaled dependent variable as a function of a suitably scaled independent variable. If scaling holds, then the data for a wide range of parameter values are said to "collapse" upon a *single* curve. To test the present data for such data collapse, we show in Fig. 3 the scaled probability density as a function of the scaled growth rates of sales and employees. The data indeed are found to collapse upon a single curve. This collapse suggests that the scaling theory might be applicable to the system of firms. The high degree of similarity in the behavior of sales and the number of employees points to the existence of large mutual correlations between sales and the number of employees. We hope to quantitatively determine these correlations in our future work.

4.2 Open Questions

We plan to complete and extend our study in order to get a comprehensive picture of the firm dynamics described in terms of conditional distribution functions. Because age has

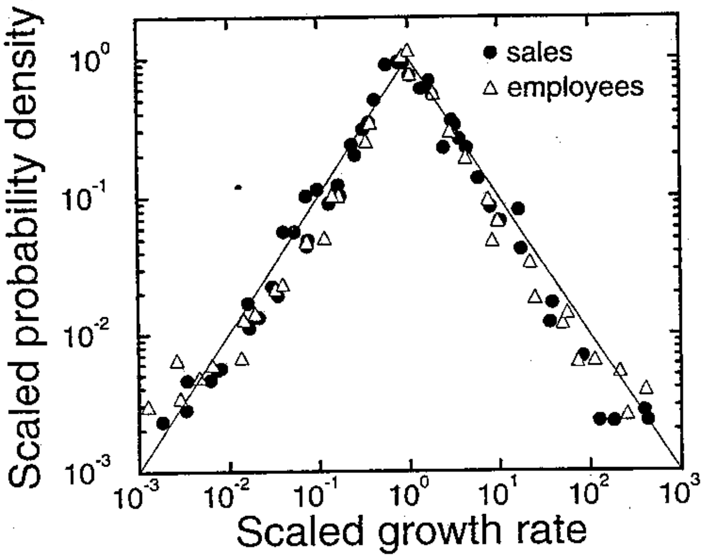

Fig. 3 Scaled probability density $p_{scal} \equiv 2^{1/2}\sigma_1(s_0)p(r_1|s_0)$ as a function of the scaled growth rate $r_{scal} \equiv 2^{1/2}|r_1 - \bar{r}_1(s_0)|/\sigma_1(s_0)$. Also we show the analogous scaled quantities for the number of employees. All the data collapse upon the universal curve $p_{scal} = \exp(-|r_{scal}|)$ as predicted by Eq. (2).

played prominently in the literature on firm dynamics,[8,14] we propose to conduct a similar study for the dependence of the growth distribution (of sales or employment) on the firm's age. Also, the change of the distribution functions over longer growth intervals than one year will be investigated and other important quantities considered, as for example the assets of the firms. The aim of these future studies is to address the following questions:

- Does the distribution of growth rates (of sales, employment, or assets) change more rapidly in time for younger than for older firms? To what extent is this influenced by the relatively smaller size of young firms?
- Is the distribution of growth rates outside the manufacturing sector similar?
- Does the shape of the distribution functions change for shorter and larger time intervals Δt? Is this shape then still independent of the initial conditions and the quantity considered, i.e., does scaling hold as exemplified in Fig. 3? What are the characteristic time scales over which the changes occur? Are the values for these time scales universal or unique to particular sectors?
- Does the exponent β change when different markets are studied? When different economies are studied?
- What model describes the time evolution of the growth distributions?

5. MODELING

A cornerstone of modern statistical physics has been the interpretation and explanation of data analysis using simple models. While these models do not precisely describe the mechanisms of a system, they do provide insight into "why" certain things occur in a complex system.

5.1 Review of Recent Results

In this section we describe some ideas on how we could possibly contribute to the understanding of firm dynamics by proposing and investigating simple models and by comparing the predictions of the models with the results of our data analysis.

The first attempt to model firm dynamics was made by Gibrat, who hypothesized that firm growth rate is independent of size.[1] He suggested that the change in sales was a random multiplicative process: The sales $S_{t+\Delta t}$ at time $t+\Delta t$ are equal to the sales S_t at time t multiplied by a random number ε_t slightly smaller or larger than one, so:

$$S_{t+\Delta t} = S_t \varepsilon_t \,, \tag{3}$$

where Δt can be much less than 1 year. After many iterations (i.e., after a long time), this stochastic process yields a *log-normal* distribution of the sales and of the annual growth rates. Therefore, it fails to explain the "tent shaped" distribution Eq. (1), obtained in our data analysis. We can try to understand Eq. (1) by assuming that successive values of S_t are correlated. Each of the firms has a tendency to maintain its initial value S_0, since it cannot adapt to large changes in demand in a short time Δt. The easiest way to incorporate these correlations is to consider a constant "back-drift," i.e.,

$$\frac{S_{t+\Delta t}}{S_t} = \begin{cases} k\varepsilon_t, & \text{for } S_t < S_0 \,, \\ \frac{1}{k}\varepsilon_t, & \text{for } S_t > S_0 \,, \end{cases} \tag{4}$$

where k is a constant larger than one. One can show that this simple process indeed yields the "tent shaped" distribution given in Eq. (1) with a width proportional to $\ln k$.

One might expect that for sufficiently long times, the distribution of growth rates approaches the distribution of the sales. Since the sales distribution can be approximated by a log-normal distribution (with the exception of very large firms), we must seek to find a model which yields a "tent shaped" distribution for small times but a log-normal distribution for large times.

A simple example of such a model can be easily constructed. We assume that each firm can be characterized by three properties: one static property and two dynamic ones. The static property is the initial sales, S_0, which is initially the same for all firms. The dynamic properties are the firm's sales and its optimal value of sales. The optimal value may be considered to be the value at which a firm maximizes its profits. There are only two rules for the model. First, the optimal value, which initially equals S_0, follows a random, multiplicative process, as suggested by Gibrat.[1] Second, the sales of the firm follow the dynamics suggested by Eq. (4) with one caveat. Instead of the "back-drift" attracting the sales toward initial sales, S_0, the sales will be attracted toward the optimal value. The growth of the optimal value is much *slower* than the growth of the sales.

Following these rules, the model produces several interesting results that correspond to the analysis in Sec. 4. Growth rates over short time intervals are characterized by a "tent-shape" distribution, Eq. (1). In contrast, growth rates over long time intervals are distributed according to a log-normal distribution.

Unfortunately, the simplification to only 2 rules appears incomplete. Although the model fits the results of our preliminary data analysis, the distribution made by the model has a

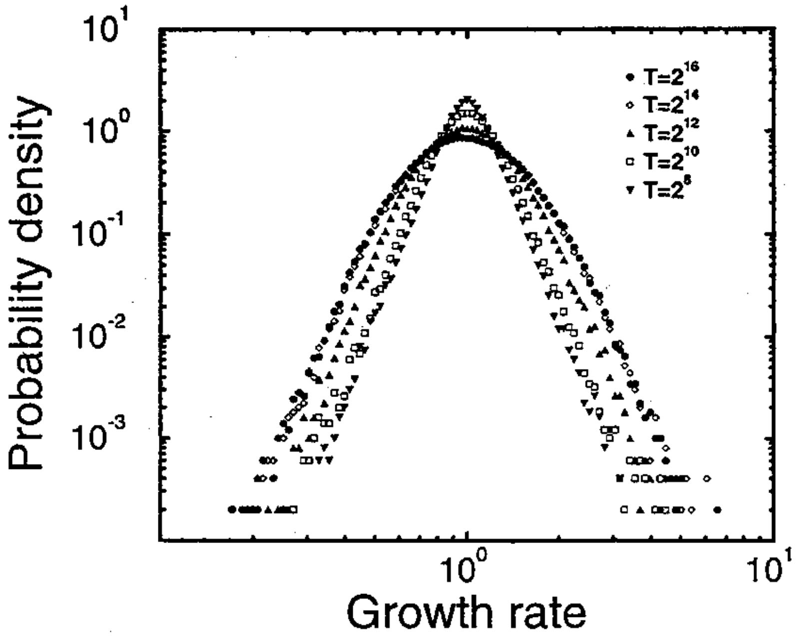

Fig. 4 Probability density of the growth rate after various computer time-steps T. For small times T, the distribution is "tent-shaped", but it approaches a stable log-normal distribution for larger T.

width that increases with time as $t^{1/2}$. However, a careful consideration of the data suggests that there is little growth in the width of the distribution over time.

Thus, we must change the model to describe the data better. It appears that the mean growth rate slightly decreases with increasing value of sales. Incorporating a linear decrease into the model, we get for long time intervals T a *stable* log-normal distribution, as shown in Fig. 4.

Up to now, we have neglected the interactions between firms, which is in many cases a reasonable approximation; for instance, to describe the size distribution of all publicly traded firms. However, for other situations such an extreme simplification is not justifiable: e.g., if we consider firms in the same industry, we obviously have to take into account the interactions and correlations between them.

The results obtained from the studies of correlations will help us to build more realistic models of firm dynamics. For example, by understanding how the growth rates of a firm are correlated from year to year we will be able to determine the time scale for the reaction of a firm to changes in its environment.

5.2 Open Questions

5.2.1 *Relation between initial size and standard deviation of growth rates*

Given that a simple relation between firm size and the standard deviation of the growth rate holds over such a wide range, that relation merits explanation. One extreme hypothesis is that modern firms consist of multiple, independent business units and that large firms consist of more units than do small firms. Based on our results, however, we can reject this hypothesis. To see this point, assume that the sales of a firm S are given by: $S = \sum_{i=1}^{N} \xi_i$, where ξ_i represents the sales of business unit i. If the ξ_i's are independently and identically distributed, then Eq. (2) follows with $\beta = 0.5$. However, we find $\beta = 0.15$, which indicates the presence of strong correlations among the firm's units.

We can try to understand this result by considering a tree-like hierarchical organization of a typical firm. The root of the tree represents the head of the company, whose policy is passed to the level beneath, and so on, until finally the units in the lowest level take action. Let us first assume that the head of the firm suggests a policy that could result in changing the sales of each division by an amount δ. If this policy is propagated through the hierarchy without any modifications, then $\beta = 0$ follows. More realistically, each unit is not only influenced by the policy of the head but also by other internal (random) factors of that unit. Then the policy of the head can be modified at each level of the firm management with a small probability. In this case a value $0 < \beta < 1/2$ can be obtained, depending on the average number of links connecting the levels. This analogy is developed elsewhere.[36]

5.2.2 *Entries, mergers, and exits*

In many of the models under study in our team, it is assumed that the number of firms acting on a given market remains constant in time. This simplification is quite useful, since it allows a better understanding of the remaining factors at play of the dynamics of firm growth. However, a more realistic approach to the problems must inevitably include the possibility of entry of new firms, the merger of existing firms, and the exit of firms from the market.

REFERENCES

1. R. Gibrat, *Les Inégalités Economiques* (Sirey, Paris, 1933).
2. P. E. Hart and S. J. Prais, *J. Roy. Stat. Soc. Series* **A119**, 150–191 (1956).
3. H. A. Simon and C. P. Bonini, *Am. Econ. Rev.* 48, 607–617 (1958).
4. Y. Ijiri and H. A. Simon, *Skew Distributions and the Sizes of Business Firms* (North Holland, Amsterdam, 1976).
5. M. H. R. Stanley, S. V. Buldyrev, S. Havlin, R. N. Mantegna, M. A. Salinger and H. E. Stanley, "Zipf plots and the size distribution of firm," *Economics Letters* **49**, 453–457 (1995).
6. S. Hymer and P. Pashigian, *J. Pol. Econ.* **52**, 556–569 (1962).
7. B. H. Hall, *J. Indust. Econ.* **35**, 583–606 (1987).
8. D. S. Evans, *J. Pol. Econ.* **95**, 657–674 (1987).
9. W. Baumol, *Business Behavior, Value, and Growth* (MacMillan, New York, 1959).
10. R. Cyert and J. March, *A Behavioral Theory of the Firm* (Prentice-Hall, Englewood Cliffs, New Jersey, 1963).
11. H. A. Simon, *Am. Econ. Rev.* **68**, 1 (1978).
12. F. Machlup, *Am. Econ. Rev.* **57**, 1 (1967).
13. M. C. Jensen and M. H. Meckling, *J. Fin. Econ.* **3**, 305–360 (1976).
14. B. Jovanovic, *Econometrica* **50**, 649–670 (1982).
15. R. R. Nelson and S. G. Winter, *An Evolving Theory of Economic Change* (Harvard University Press, Cambridge, Massachusetts, 1982).
16. B. R. Holmstrom and J. Tirole, in *Handbook of Industrial Organization* Vol. 1, eds. R. Schmalensee and R. Willig (North Holland, Amsterdam, 1989), pp. 61–133.
17. O. E. Williamson, in *Handbook of Industrial Organization* Vol. 1, eds. R. Schmalensee and R. Willig (North Holland, Amsterdam, 1989), pp. 135–182.
18. P. Milgrom and J. Roberts, *Economics, Organization, and Management* (Prentice-Hall, Englewood Cliffs, New Jersey, 1992).
19. R. Lucas, *Bell J. Econ.* **9**, 508–523 (1978).

20. R. Solow, *Review Econ. Stat.* **39**, 312 (1957).
21. M. I. Kamien and N. L. Schwartz, *Market Structure and Innovation* (Cambridge University Press, Cambridge, 1982).
22. W. M. Cohen and R. C. Levin, in *Handbook of Industrial Organization* Vol. 2, eds. R. Schmalensee and R. Willig (North Holland, Amsterdam, 1989), p. 1059.
23. S. R. Fazzari, R. G. Hubbard and B. Petersen, *Microeconomics Brookings Papers on Economic Activity*, 141 (1988).
24. S. C. Myers and N. Majluf, *J. Fin. Econ.* **13**, 147 (1984).
25. R. G. Hubbard, (ed.), *Asymmetric Information, Corporate Finance and Investment* (University of Chicago Press, Chicago, 1990).
26. K. A. Froot, D. S. Scharfstein and J. C. Stein, *Harvard Business Review*, 91 (1994).
27. M. C. Jensen, *Am. Econ. Rev.* **76**, 323 (1986).
28. M. H. R. Stanley, L. A. N. Amaral, S. V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M. A. Salinger and H. E. Stanley, "Scaling behavior in the growth of companies," *Nature* **379**, 804–806 (1996).
29. R. N. Mantegna and H. E. Stanley, *Nature* **376**, 46–49 (1995).
30. H. E. Stanley, *Introduction to Phase Transitions and Critical Phenomena* (Oxford University Press, New York, 1971).
31. A. Bunde and S. Havlin (eds.), *Fractals in Science* (Springer-Verlag, Berlin, 1994).
32. A. L. Barabasi and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, 1995).
33. S. V. Buldyrev, P. Garik, S. Milosevic, H. E. Stanley, E. F. Taylor and P. A. Trunfio, *Fractals in Science* (Springer Verlag, Berlin and New York, 1994).
34. R. F. Engle, *Econometrica* **50**, 987–1007 (1982).
35. T. Bollerslev, R. Y. Chou and K. F. Kroner, *J. Economet.* **52**, 5–59 (1992).
36. See article of H. E. Stanley et al. in this volume.
37. S. V. Buldyrev, L. A. N. Amaral, S. Havlin, H. Leschhorn, P. Maass, M. A. Salinger, H. E. Stanley and M. H. R. Stanley, "Scaling behavior in economics: Empirical results and modeling," (preprint).