



Long term memory in extreme returns of financial time series

Lev Muchnik ^a, Armin Bunde ^b, Shlomo Havlin ^{c,*}

^a Leonard N. Stern School of Business, New York University, United States

^b Institut für Theoretische Physik III, Justus-Liebig-Universität, D-35392 Giessen, Germany

^c Minerva Center and Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel

ARTICLE INFO

Article history:

Received 15 February 2009

Received in revised form 15 May 2009

Available online 6 June 2009

PACS:

05.40.-a

02.50.-r

05.45.Tp

89.65.Gh

Keywords:

Extreme values

Long-term correlation

Long-term memory

Volatility

Econophysics

ABSTRACT

It is well known that while daily price returns of financial markets are uncorrelated, their absolute values ('volatility') are long-term correlated. Here we provide evidence that certain subsequences of the returns themselves also exhibit long-term memory. These subsequences consist of maxima (or minima) of returns in consecutive time windows of R days. Our analysis shows that for both stocks and currency exchange rates, long-term correlations are significant for $R \geq 4$. We argue that this long-term memory which is similar to that observed in volatility clustering sheds further insight on price dynamics that might be used for risk estimation.

Published by Elsevier B.V.

The belief that financial time series exhibit random walk like behavior was already postulated by Louis Bachelier back in 1900 [1] and has been deeply rooted in the vast majority of economical models since then. For example, in 1970s this approach led to the emergence of the Efficient Market Hypothesis [2,3] (EMH) which is still considered to be one of the major pillars of economics. Approximately at the same time, the first attempts to challenge the random walk assumption were made [4,5]. Since then, the attempts to identify and classify general deviations of financial time series from the Wiener process became more systematic due to the availability of detailed financial data and the evolution of statistical analysis methods in physics (such as fractal methods [6] and random matrix theory [7]) and in economics (mainly technical analysis [8]).

A considerable amount of empirical research has been invested in exploring deviations and their origin from this paradigm [9], which are now usually referred to as 'stylized facts' [10]. Some of them can be associated with or even derived from irrational patterns in human behavior [11–13], others are related to trading arrangements. However, the origin of the majority of the stylized facts remains unclear. Moreover, the greater part of these artifacts is not persistent, and either show up in very special conditions or disappear soon after their discovery and publication [14]. This emphasizes the importance of the remaining stylized facts where their robustness suggests that they are inherent to financial market dynamics.

Perhaps the most intriguing family of stylized facts is associated with the presence of long-term correlations in financial data. It is clear (and has been shown repeatedly) that price returns or any other time series offshoot cannot be persistently correlated over some reasonable range of time (currently order of few minutes) to the level where it can be exploited to gain excess returns [15–17]. Otherwise this fact could easily be exploited for price moves prediction that would eliminate the correlations. Existing short range correlations in returns are frequently related to the tendency of investors to underreact

* Corresponding author.

E-mail address: havlin@ophir.ph.biu.ac.il (S. Havlin).

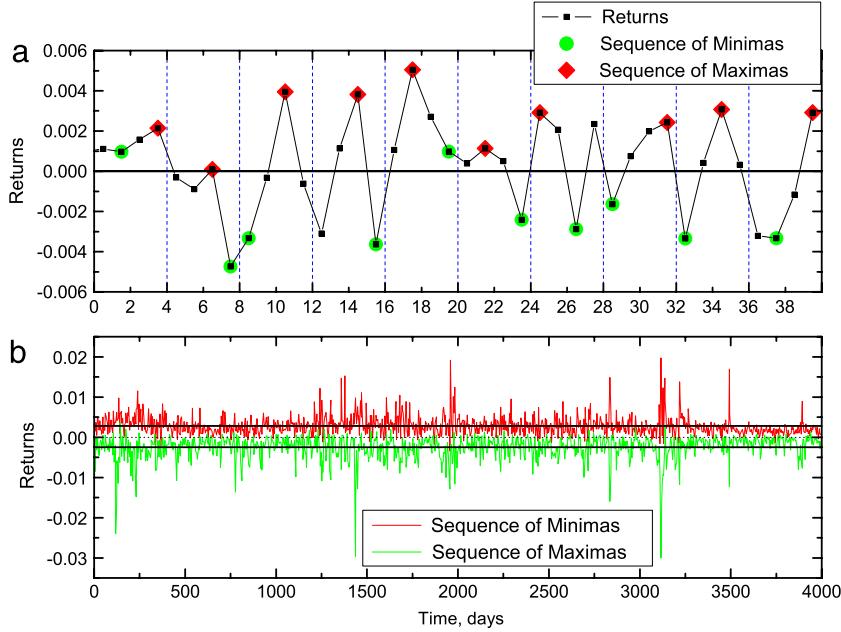


Fig. 1. (color online) (a) Illustrates the generation of maxima (diamonds) and minima (circles) time series in the sequence of the S&P500 daily returns for the interval $R = 4$. (b) Shows the maxima (red online) and minima (green online) obtained for $R = 4$ over 4000 days.

to incoming information creating some measurable momentum [18]. Hence, one cannot expect to have easily exploitable correlation patterns in financial data sets. This does not mean, however, that correlations do not exist at all.

In fact, one of the most exciting and mysterious properties of financial time series is the presence of long-term correlation patterns in the variance of returns known as volatility clustering [7,19–28].

Here we show that even in the return records there are subsequences that are long-term correlated and add further insight into the market dynamics. These subsequences consist of the maxima or the minima of the returns, obtained in successive time intervals of fixed length R . Both extreme sequences exhibit a pronounced long-term persistence in a way that can be compared with the volatility persistence, with a correlation exponent comparable to that of the volatility.

We focus on the daily logarithmic returns ("returns" hereafter) r_i and the daily volatility v_i . In the current manuscript we choose to use the following definitions: $r_i = \ln [P_{i+1}/P_i]$ and $v_i = |r_i|$ where P_i is the asset closing price at day i . These definitions are trivially generalized to returns and volatility computed for any arbitrary time interval. Long-term correlated records $\{x_i\}$, $i = 1, \dots, N$, with zero mean and unit variance, are characterized by an autocorrelation function $C_x(s) = \langle x_i x_{i+s} \rangle \equiv 1/(N-s) \cdot \sum_{i=1}^{N-s} x_i x_{i+s}$ that decays by a power law, $C_x(s) \sim s^{-\gamma}$, where the correlation exponent γ is between 0 and 1. To test for long-term correlations, it is also useful to employ the detrended fluctuation analysis (DFA2) [29–31]. In DFA2, one considers the cumulated sum ('profile') $Y_i = \sum_{j=1}^i x_j$ and studies, in time windows of length s , the mean fluctuations $F(s)$ of the profile around the best quadratic fit. For long-term correlated data, $F(s)$ scales as $F(s) \sim s^\alpha$, with $\alpha = 1 - \gamma/2$ [29].

Fig. 1(a) shows a typical example of a financial time series (S&P 500) for 40 sequential days. The figure also shows the sequence $m_R(t)$ of maxima (and minima) of the data in windows of length $R = 4$, which are constructed from the original sequence of daily logarithmic price returns, r_i , by selecting maximal (minimal) values of daily returns over each interval of length R . Obviously, both subsequences contain R times fewer points than the original ones. **Fig. 1(b)** shows typical sequences of these maxima and minima for $R = 4$. Two solid horizontal lines in **Fig. 1(b)** represent the mean maxima and minima, respectively. In both figures, one can see pronounced patches of small and large maxima and resp. minima (above and below the mean) that are clumped together. The patches in **Fig. 1(b)** demonstrate qualitatively the occurrence of a memory effect (clustering), where large maxima (minima) tend to follow large maxima (minima) and small maxima (minima) tend to follow small ones.

The clustering of extremes is qualitatively different from the volatility clustering and yield further information by distinguishing between gains (maxima) and losses (minima). The clustering of gains indicates that large gains within intervals of R days are likely to be followed by large gains within following intervals. Likewise occurs for losses.

To quantify this memory effect, we have analyzed the daily maxima and minima 43 to 55 years long records of seven traded commodities (S&P 500 index and 6 stocks: IBM, DuPont, AT&T, Kodak, GE, Coca-Cola) and seven 32 to 34 years long currency exchange rates (US Dollar vs. Japanese Yen, British Pound vs. Swiss Franc, US Dollar vs. Swedish Krona, Danish Krone vs. Australian Dollar, Danish Krone vs. Norwegian Krone, US Dollar vs. Canadian Dollar and US Dollar vs. South African Rand) obtained, respectively, from *Yahoo!Finance* [32] and US Federal Reserve [33]. Here we present the results for the IBM stock, the S&P500 index and the exchange rates of the Japanese Yen and the Canadian Dollar against the US Dollar. These selected

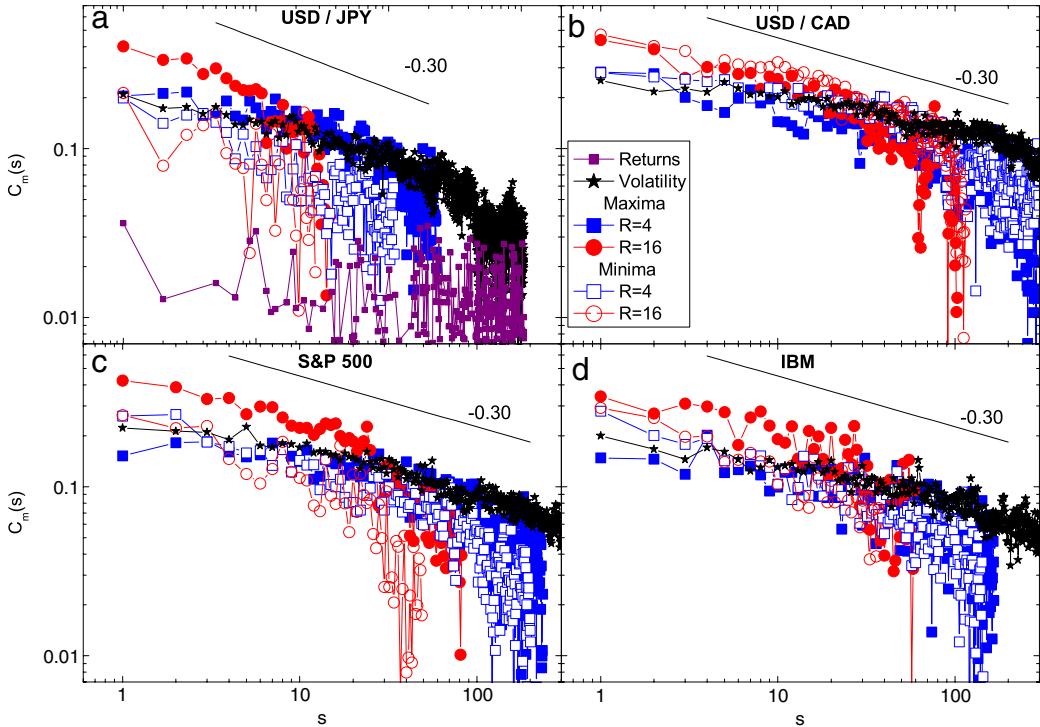


Fig. 2. (color online) Autocorrelation function for the minima and maxima sequences of returns of two currency exchange rates (a) USD vs. Yen, and (b) USD vs. Canadian Dollar, and two stock records (c) the S&P 500 index and (d) the IBM record. Results for two different intervals (R) are shown. The autocorrelation function of the volatility and the returns (only in (a)) are also shown for comparison.

assets are quite representative and our results are confirmed also for the other records. We choose to study daily data records to avoid intraday trends and focus on long-term effects.

We first study the autocorrelation function $C_x(s)$, where the parameter x now stands for the sequence of maxima (resp. minima) in the record for a given window length R , as defined in Fig. 1(a). Fig. 2 shows, for the four assets considered, the autocorrelation function of the minima and maxima for two sequel sizes, $R = 4$ and $R = 16$, as a function of s .

For comparison, the 4 panels also show the respective autocorrelation functions of the volatility. The figure shows clearly that the autocorrelation functions of the minima and maxima decay by a power law, with an exponent γ close to 0.3 in all four examined records. The figure also shows that their decay is quite similar to the decay of the volatility autocorrelation function and very different from the autocorrelation function of the returns (which is zero at those time scales [2,23,34,35]). For comparison, we also show the autocorrelation function of the returns (corresponding to $R = 1$) for the USD/CAD exchange rate and the IBM record. Since the autocorrelation function is known to be highly susceptible to nonstationarities and temporal trends in the data, we also employed DFA2, which can overcome linear trends in the data [29,30]. The results, presented in Fig. 3, are a further indication that long-term correlation patterns occur in both minima and maxima records of the returns.

To estimate the degree to which the information contained in the sequences of the extremes is related to the information contained in the corresponding volatility sequence, we have computed the cross-correlation coefficients between the maxima and the corresponding volatility, the minima and the corresponding volatility, as well as between the sequences of maxima and minima. The result for $R = 4$ and 16 is shown in Fig. 4(a)–(d) for 6 stock records and 13 exchange rates (against the US-Dollar). All four figures show similar correlations for different stocks and currencies. For $R = 4$, almost no correlation between maxima and minima exists while the correlations between each of them and the volatility is rather high. For higher value of R ($R = 16$), all correlations are approximately similar (around 0.3–0.5). Fig. 4(e) and (f) show that the correlation coefficients depend strongly on the interval length R . For $R < 5$ we observe strong anticorrelations between minima and maxima which disappear with increasing R . While increasing R , the correlation coefficient between minima and maxima increases monotonically from negative values (anticorrelations) towards values close to 0.4, while for the correlations between minima/maxima and the volatility the coefficient decreases, from close to 1 for $R = 1$ to close to 0.4 for large interval lengths. The disappearance of anticorrelations between minima and maxima is probably due to the decay of autocorrelation in volatility as R increases.

Fig. 4 suggests that while there is some relation between the information contained in the volatility and the sequences of minima and maxima, it is not trivial. This is since the correlation changes depending on the interval R and the values of the correlation are not sufficiently high to indicate equivalency between the two different approaches.

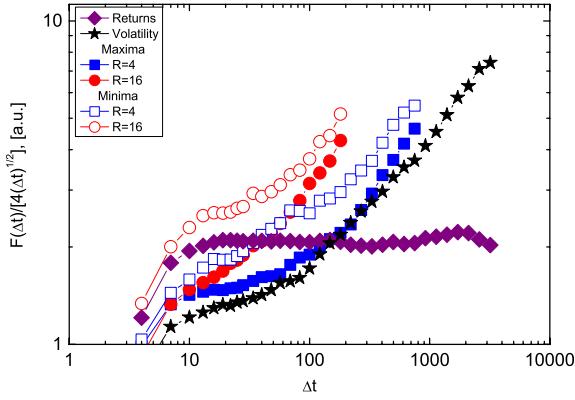


Fig. 3. (color online) Detrended fluctuation analysis (DFA2) of returns, volatility, minima and maxima records for the S&P500 index, for $R = 4$ and $R = 16$. The units along the y -axis are arbitrary. The asymptotic slope is close to 0.35, i.e. $\alpha = 0.85$, in agreement with the correlation exponent γ about 0.3 obtained from Fig. 2. Returns and volatility graphs are computed for intervals of $R = 4$ and based on the return of S&P 500 between the closing prices of the first and the last days of the interval, $r_i = \ln[P_{i+R}/P_i]$ and correspondingly $v_i = |r_i|$.

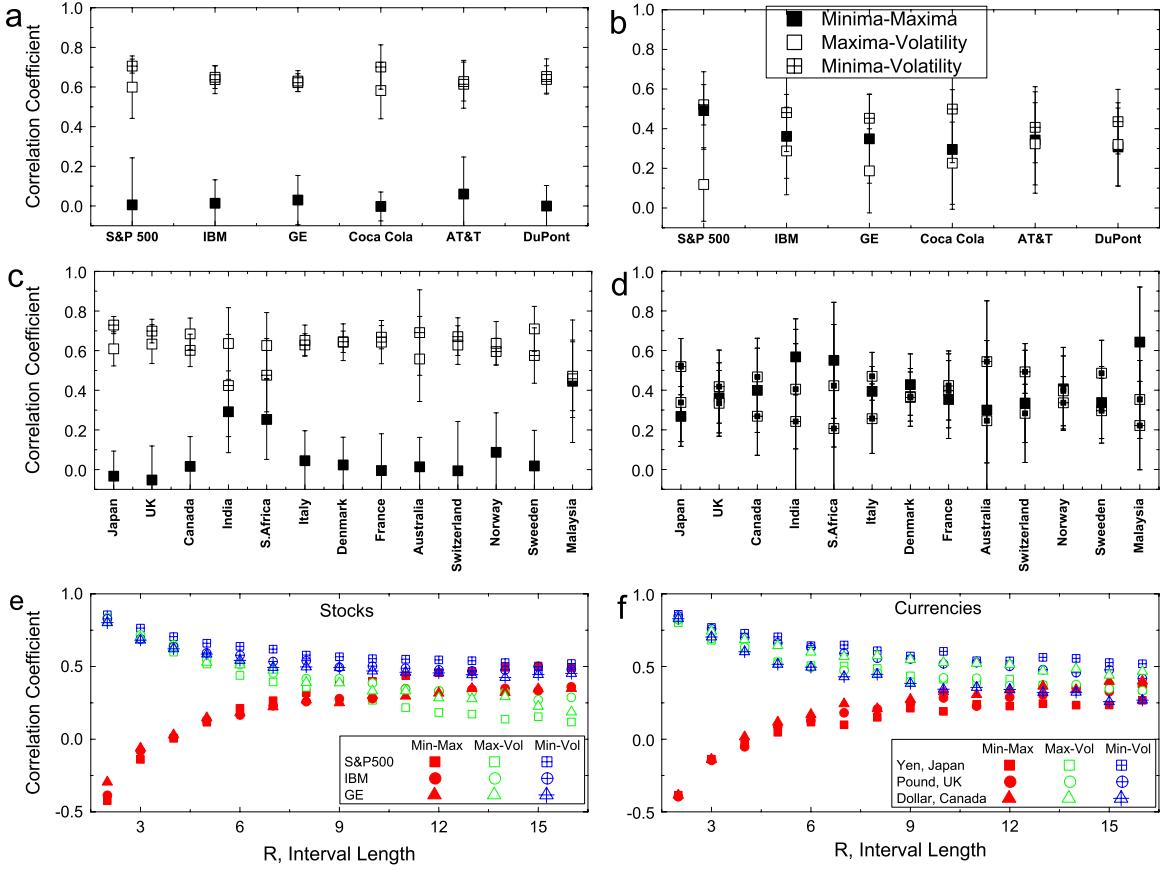


Fig. 4. (color online) Coefficients of cross-correlation between the sequences of extremes (Minima and Maxima) and volatility for (a) Stocks with $R = 4$, (b) Stocks with $R = 16$, (c) Currencies with $R = 4$ and (d) Currencies $R = 16$. The subplots (e) and (f) show dependence of the correlation coefficient on the interval R for three typical stocks (e) and currencies (f). To simplify the comparison between the correlation of minima and volatility vs. the maxima and volatility, in all cases we show the absolute of the correlation coefficient, while between minima and maxima we show the actual value of the correlation coefficient. Error bars in plots (a)–(d) represent the standard deviation obtained by splitting the dataset into 10 equal subsequences.

To further quantify the memory effect, we study the conditional average m_R (m_0) of those maxima and minima values that immediately follow a value of m_0 . To improve the statistics, we have binned the data into eight bins. First, we sorted the full data set of the maxima (resp. minima) in increasing (decreasing in the case of minima) order and divided it into eight subrecords (bins) O_1, O_2, \dots, O_8 such that each subrecord contains 1/8 of the total number of maxima (resp. minima).

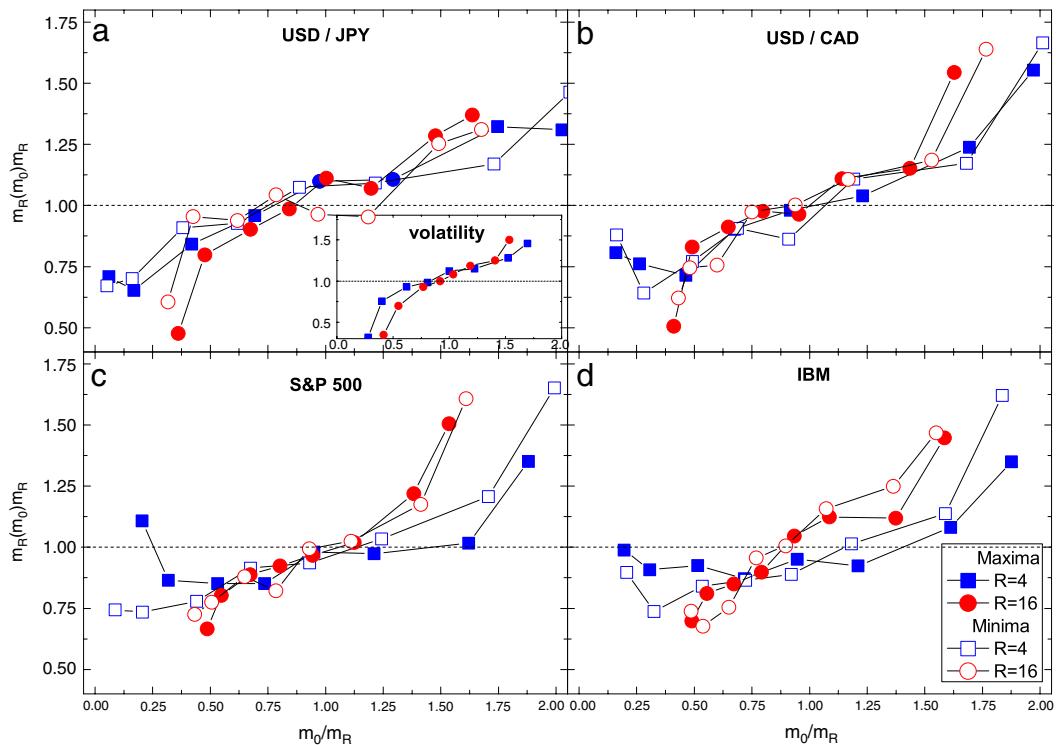


Fig. 5. (color online) Mean conditional minima and maxima $m_R(m_0)$ of returns as a function of the corresponding extreme value m_0 in the preceding interval: (a) USD vs. Japanese Yen, (b) USD vs. Canadian Dollar, (c) the S&P500 index, and (d) the IBM record for $R = 4$ and $R = 16$. The inset in (a) shows the corresponding conditional volatility.

Therefore, the $N/8$ smallest maxima (resp. largest minima) are in O_1 , while the $N/8$ largest (and correspondingly, smallest minima) values are in O_8 . By choosing m_0 to be within octaves, we actually keep m_0/m_R constant for different values of R (here m_R is the unconditional average). Fig. 5 shows $m_R(m_0)/m_R$ as a function of m_0/m_R . Because of the strong memory in the maxima and minima records, $m_R(m_0)/m_R$ is well below 1 for $m_0 \in O_1$ and respectively well above 1 for $m_0 \in O_8$. This effect of memory is absent (not shown here) when we shuffle either the record of the returns or the records of the extremes. By shuffling, the memory is removed and $m_R(m_0)/m_R$ is equal to 1 for all m_0/m_R values. Note that for different R values and for both sequences of minima and maxima, $m_R(m_0)/m_R$ scales with m_0/m_R . Fig. 5 reveals that a period with a low maximum (where the gains are low) has a higher chance to be followed by periods of low gains than by high gains. Similarly, a period with a high minimum (where no or low losses occur) will be followed, with higher probability, by periods of low losses.

The memory patterns for the extreme returns shown here actually address the question of market predictability. We showed that this memory yield further insight on price returns that might be used in practice for risk estimation. The origin of the long-term memory in both the extreme returns and the volatility is still unclear [36]. It is interesting to note that the long-term correlations found in the volatility and in extreme returns are quite similar to those found in other complex systems, for example in temperature records [37], river flows [38,39], DNA [40], heart-beat intervals [41], or earthquakes [42]. Here also the mechanism for the long term memory is not yet known, but might be of similar origin. Further study of this phenomenon may lead to a deeper understanding of the general mechanisms governing long term memory in complex dynamical systems.

References

- [1] L. Bachelier, Théorie de la spéculation, Annales Scientifiques de l’École Normale Supérieure Sér. 3 (17) (1900) 21–86.
- [2] E.F. Fama, Efficient capital markets: A review of theory and empirical work, Journal of Finance 25 (2) (1970) 383–417.
- [3] B.G. Malkiel, A Random Walk Down Wall Street, W. W. Norton & Co., New York, 1973.
- [4] P. Cootner (Ed.), The Random Character of Stock Market Prices, M.I.T. Press, Cambridge, MA, 1964.
- [5] B.B. Mandelbrot, Journal of Political Economy 71 (1963) 420–440.
- [6] R.N. Mantegna, H.E. Stanley, An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge University Press, 1999.
- [7] V. Plerou, et al., Universal and nonuniversal properties of cross correlations in financial time series, Physical Review Letters 83 (7) (1999) 1471–1474.
- [8] J.J. Murphy, Technical Analysis of Futures Markets, Institute of Finance, New York, 1986.
- [9] A.W. Lo, A.C. MacKinlay, A Non-Random Walk Down Wall Street, Princeton University Press, 1999, pp. 448.
- [10] R. Cont, Empirical properties of asset returns: Stylized facts and statistical issues, Quantitative Finance 1 (2) (2001) 223–236.
- [11] C.F. Camerer, G. Loewenstein, R. Rabin, Advances in Behavioral Economics (2003).
- [12] A. Shleifer, Inefficient Markets: An Introduction to Behavioral Finance, Oxford University Press, 1999.
- [13] D. Hirshleifer, Investor psychology and asset pricing, Journal of Finance, American Finance Association 56 (4) (2001) 1533–1597. 08.
- [14] W.G. Schwert, Anomalies and market efficiency, in: G.C. al (Ed.), Handbook of the Economics of Finance, North Holland, 2001.

- [15] T. Odean, Do investors trade too much? *American Economic Review* 89 (1999) 1279–1298.
- [16] Y. Liu, et al., Statistical properties of the volatility of price fluctuations, *Physical Review E* 60 (1999) 1390.
- [17] D. Lesmond, M. Schill, Z. Chunsheng, The illusory nature of momentum profits, Tulane University, 2001 (unpublished manuscript).
- [18] B.G. Malkiel, The efficient market hypothesis and its critics, *Journal of Economic Perspectives* 17 (1) (2003) 59–82.
- [19] R.A. Wood, T.H. McInish, J.K. Ord, An investigation of transactions data for NYSE stocks, *Journal of Finance* 40 (3) (1985) 723–739.
- [20] L. Harris, A transaction data study of weekly and intradaily patterns in stock returns, *Journal of Financial Economics* 16 (1) (1986) 99–117.
- [21] A. Admati, P. Pfeiferer, A theory of intraday patterns: Volume and price variability, *The Review of Financial Studies* 1 (1) (1988) 3–40.
- [22] G.W. Schwert, Why does stock market volatility change over time? *Journal of Finance* 44 (5) (1989) 1115–1153.
- [23] Y. Liu, et al., Correlations in economic time series, *Physica A* 245 (3–4) (1997) 437–440.
- [24] P. Cizeau, et al., Volatility distribution in the S&P500 stock index, *Physica A* 245 (3–4) (1997) 441–445.
- [25] K. Yamasaki, et al., Scaling and memory in volatility return intervals in financial markets, *Proceedings of the National Academy of Sciences of the United States of America* 102 (26) (2005) 9424–9428.
- [26] V. Plerou, et al., Price fluctuations, market activity and trading volume, *Quantitative Finance* 1 (2001) 262.
- [27] V. Plerou, P. Gopikrishnan, H.E. Stanley, Quantifying fluctuations in market liquidity: Analysis of the bid-ask spread, *Physical Review E* 71 (2005) 046131.
- [28] F. Wang, et al., Indication of multiscaling in the volatility return intervals of stock markets, *Physical Review E* 77 (2008) 016109.
- [29] C.-K. Peng, et al., Mosaic organization of DNA nucleotides, *Physical Review E* 49 (2) (1994) 1685–1689.
- [30] A. Bunde, et al., Correlated and uncorrelated regions in heart-rate fluctuations during sleep, *Physical Review E* 85 (17) (2000) 3736–3739.
- [31] J.W. Kantelhardt, et al., Detecting long-range correlations with detrended fluctuation analysis, *Physica A* 295 (2001) 441–454.
- [32] Yahoo!Finance. <http://finance.yahoo.com>.
- [33] Federal Reserve System. www.federalreserve.gov/releases/H10/hist.
- [34] Z. Ding, C.W.J. Granger, R.F. Engle, *Journal of Empirical Finance* 1 (1993) 83.
- [35] M.M. Dacorogna, et al., *Journal of International Money and Finance* 12 (1993) 413.
- [36] T. Kalisky, Y. Ashkenazy, S. Havlin, Volatility of linear and nonlinear time series, *Physical Review E* 72 (1) (2005).
- [37] E. Koscielny-Bunde, et al., Indication of a universal persistence law governing atmospheric variability, *Physical Review Letters* 81 (3) (1998) 729–732.
- [38] V.N. Livina, et al., Nonlinear volatility of river flux fluctuations, *Physical Review E* 67 67 (2003) 042101.
- [39] J.F. Eichner, et al., Extreme value statistics in records with long-term persistence, *Physical Review E* 73 (1) (2006) 016130.
- [40] C.-K. Peng, et al., Long-range correlations in nucleotide sequences, *Nature* 356 (1992) 168–170.
- [41] C.-K. Peng, et al., Long-range anticorrelations and non-Gaussian behavior of the heartbeat, *Physical Review Letters* 70 (1993) 1343–1346.
- [42] V.N. Livina, S. Havlin, A. Bunde, Memory in the occurrence of earthquakes, *Physical Review Letters* 95 (2005) 208501.