

Multiple metastable network states in urban traffic

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While abrupt regime shifts between different metastable states have occurred in natural systems from many areas including ecology, biology, and climate, evidence for this phenomenon in transportation systems has been rarely observed so far. This limitation might be rooted in the fact that we lack methods to identify and analyze possible multiple states that could emerge at scales of the entire traffic network. Here, using percolation approaches, we observe such a metastable regime in traffic systems. In particular, we find multiple metastable network states, corresponding to varying levels of traffic performance, which recur over different days. Based on high-resolution global positioning system (GPS) datasets of urban traffic in the megacities of Beijing and Shanghai (each with over 50,000 road segments), we find evidence supporting the existence of tipping points separating three regimes: a global functional regime and a metastable hysteresis-like regime, followed by a global collapsed regime. We can determine the intrinsic critical points where the metastable hysteresis-like regime begins and ends and show that these critical points are very similar across different days. Our findings provide a better understanding of traffic resilience patterns and could be useful for designing early warning signals for traffic resilience management and, potentially, other complex systems.

resilience | urban traffic | percolation | multiple states | tipping point

nderstanding the intricate patterns of city traffic is a major Understanding the intreate patterns of end applications. In challenge, in terms of both theory and applications. In particular, severe congestion in urban transportation can cause substantial damage to economic and other systems, thus highlighting the importance of studying and understanding the complex dynamics of traffic. One significant question that has so far been unknown is whether urban traffic exhibits multiple metastable macroscopic states, which would suggest different strategies for managing the system (1). In general, multiple states have been found in many natural systems including ecology and climate systems (2). These states signify the system resilience since these natural systems can respond differently when facing the same level of perturbations. This metastability is due to the fact that these systems undergo a critical transition between multiple states, where a small disturbance may lead to an abrupt regime shift (3, 4). The fundamental difficulty in identifying such multiple states is related to the lack of methods capable of characterizing urban traffic at a global level.

Most existing resilience studies focus on the critical transition between multiple states in natural systems (5), where either a single temporal parameter (6) or experimental conditions (7) demonstrate the approach to a critical point, representing an upcoming system phase transition. Due to the nonstationary time series and long-range spatial correlations during system evolution, it becomes difficult to apply simply existing resilience analysis (8) to the transportation system, as well as other critical infrastructures. For example, although autocorrelation predicts the closeness to a fold bifurcation (5), noisy measurements and nonstationarity in the time series in many realistic cases may mask the correlations and lead to false predictions. To overcome this issue, spatial patterns with more information and higher resolution could be useful for predictions at short time scales and even provide possible mitigation methods. We study here the spatial patterns of functional traffic networks (consisting of connected roads with relatively high speed, called "functional") in cities and ask whether the functional network exhibits multiple metastable states under a specific congestion rate in the road network. Using a network metric (the size of the largest functional cluster) based on percolation theory, we find metastable states with a hysteresis-like phenomenon in urban traffic. This traffic metastability phenomenon can be observed in different traffic datasets. Furthermore, we also identify the tipping points for the onset of the multiple states, where the system can change between them. These findings not only reflect the system's operational mechanism, but also could provide city managers with early warning signals of imminent major collapse.

The urban transportation network can be represented as a network structure with intersections as nodes and road segments as links. In this study, we have collected real-time traffic data (instantaneous speed in each road) during a specific period (e.g., 1 mo) for several cities in China. These velocities are recorded by floating cars at a resolution of 1 min. Due to the flow dynamics, the relationship between system reliability and structural

Significance

Different traffic states experienced by commuters may be due to localized random jams or overall network states, which is decided by system response to perturbations. While for natural systems, the existence of multiple states is common, the evidence for multiple states in transportation has so far been unclear, which hinders our ability to predict and prevent the spread of traffic jams. Based on percolation theory and high-resolution GPS datasets, we find strong evidence for the existence of multiple metastable network states corresponding to distinct system performance regimes. Our results may provide deep insight for a city traffic manager seeking to design early warning signals to prevent the system from shifting to a state of severe congestion.

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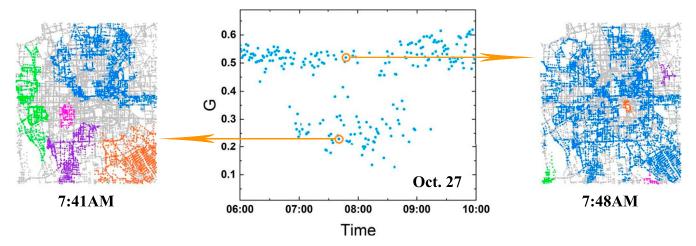


Fig. 1. Examples of metastable states in urban traffic for a typical day based on real data of Beijing. (*Center*) Relative size of the largest functional cluster, represented by G, evolves with time between essentially two states during morning hours in a typical day (October 27) for a given congestion rate f = 0.25. (*Right* and *Left*) A clear bistate phenomenon can be seen: The five largest functional clusters in the urban traffic at certain times of October 27 (which are colored in blue, orange, purple, green, and magenta from large to small) represent two typical traffic states (for samples in other days, see *SI Appendix*, Fig. S2-2).

connectivity is not always intuitive (9, 10). This traffic system which combines dynamics (traffic flow) and structure (road network) can then be analyzed by percolation theory for its global performance under different failure scenarios (congestion rates) (11, 12). From the viewpoint of reliability management, congestion can be regarded as the failure of the traffic system. A road with velocity lower than a specific service level (such as a given relative velocity threshold) is considered failed since it can hardly handle traffic flows within a certain time. On the contrary, roads with higher speed can be regarded as functional. Under an increasing traffic load, a fraction of roads in a city will be regarded as failed (below the relative velocity threshold), while the remaining free-flow roads can form connected functional clusters of different sizes. These clusters of connected functional roads reflect the accessibility of the road network. The network accessibility is defined by the functional fraction of the traffic network, i.e., the relative size of the largest functional cluster, above a certain service requirement (12-14). For a given threshold, the greater the largest functional cluster is, the higher is the accessibility of the city traffic. While prior studies explain how local blockages of traffic flow can result in global congestion, here we study the fundamental question of whether, as a feature of system resilience, the global traffic flow under the same fraction of perturbations can display multiple network states.

Results

Our datasets have been collected from real-time traffic in the (directed) road network of different cities. Beijing includes over 27,000 nodes and 52,000 links (within the 5th Ring Road). The velocity dataset covers real-time velocity records of roads in Beijing for 17 working days in October 2015 and 9 working days in October 2018. Road velocity is obtained from global positioning system (GPS) data recorded in floating cars with 1-min resolution. The dataset of Shanghai, which is composed of over 26,000 nodes and 51,000 links, has the same temporal resolution as Beijing and covers 5 working days in October 2015 and 17 working days in October 2017. Jinan, a comparatively small city in China, has around 12,000 nodes and 22,000 links. The velocity data are also recorded with the same resolution, covering 5 working days in October 2018.

By applying percolation analysis for each city, we first determine the service level of each road in the city. Here we use the relative velocity derived from real-time data to indicate the road operation level. For a given road, we sort in increasing order the velocity measured at a given day and choose the 95th percentile as the standard maximal velocity. Then we normalize the velocities of each road at each instant by dividing them by their standard maximal velocity on that day. In this way we obtain the relative velocity of each road at every minute, $r_{ij}(t) = v_{ij}(t)/v_{ij}^m$. Here, $v_{ij}(t)$ is the real-time velocity of the road connecting node *i* to node *j* at time *t*, v_{ij}^m is the standard maximal velocity of this road for that day, and $r_{ij}(t)$ is the relative velocity. For a given relative velocity threshold, *q*, the road with $r_{ij}(t) < q$ is considered failed (10). The congestion rate, *f*, is therefore defined as the fraction of failed roads with respect to the total number of roads.

Here we mainly focus on the transportation system of Beijing in China, as an ideal system for our study. In Beijing with the central area covering over 700 km² and a population of over 20 million inhabitants, passengers usually spend almost twice as much time for commutes during rush hours, compared to nonrush hours*, which causes huge economic losses and other social risks. We study data from the month of October which is one of the busiest months in Beijing. As shown in Fig. 1, we observe that the urban traffic can have multiple network states describing the global performance at the scale of the entire city. For instance, for a given fraction (i.e., f = 0.25) of congested roads, at 7:41 AM of 27 October 2015, the traffic network breaks into several fragmented parts; however, a few minutes later, with the same fraction of congested roads, at 7:48 AM, these small traffic clusters merge into a large functional cluster spanning almost the whole city of Beijing. In fact, during most morning hours in a typical day, the dynamical state of the traffic system in Fig. 1 shifts back and forth frequently between these two states (more demonstrations are shown in SI Appendix, Fig. S2-2). This regime shift suggests that urban traffic may undergo a sharp transition between alternative network states.

To further study the multiple states in urban traffic, we analyze the entire dataset and measure how the size of the largest functional cluster (G) changes with the congestion rate (f) (Methods). As seen in Fig. 2A, the largest functional cluster decreases as the congestion rate f increases. More importantly, at a given congestion rate f, urban traffic can form either a large functional network (we call it an H state, representing a high-performance

^{*}http://huiyan.baidu.com/cms/report/2018Q2jiaotong/index.html.

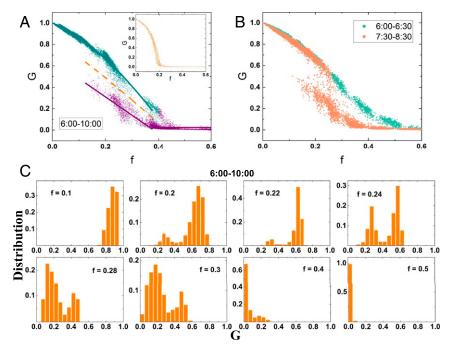


Fig. 2. Multiple states in real-time traffic percolation. (*A*) In real time, the percolation process shows a metastability phenomenon: There exist essentially two states of city traffic for a given f value (between 0.1 and 0.4) for the morning hours (during 6:00 to 10:00 AM) in all of the 17 working days in October 2015. For comparison, we observe (*Inset*) only one state in a controlled random case where links are removed randomly. (*B*) The *G*-f plot for different periods (nonrush hours during 6:00 to 6:30 AM and rush hours during 7:30 to 8:30 AM) in all of the working days. (*C*) The distribution of *G* for different values of *f*. The bistate phenomenon is clearly seen between f = 0.2 and f = 0.3.

state of the traffic network) or a much smaller network (we call it an L state, representing a low-performance state of the traffic network). This metastability is surprising since the largest cluster of the structural road network decreases monotonically as the removal fraction increases as seen for a controlled random percolation process, in Fig. 2A, Inset, with a critical point around $f_c = 0.2$. This behavior can be regarded as the structural reliability of the road network, neglecting the dynamics of traffic flow interactions. Note that in the city of Shanghai (SI Appendix, Figs. S1-1 and S1-2), there seem to be even three regimes rather than the two regimes found in Beijing. We argue that the landscape of the attraction basin in urban traffic depends on both the city network structure and the flow dynamics therein. We have studied (SI Appendix, Figs. S1-3 and S1-4) a different time period in Beijing (i.e., October 2018), which also demonstrates similar multiple states. However, for Jinan, which is smaller compared to Beijing and Shanghai, our results suggest that only a single state is observed (SI Appendix, Figs. S1-3 and S1-4).

The phenomenon of multiple states often occurs at a specific time window. For example, in nonrush hours during 6:00 to 6:30 AM, the percolation is monotonically decreasing with increasing failure fraction (Fig. 2B) (see also SI Appendix, Fig. S1-5 for more details). Note that this percolation process is significantly more robust ($f_c \approx 0.5$) than the structurally controlled case in the inset of Fig. 2A ($f_c \approx 0.2$). On the other hand, for rush hours during 7:30 to 8:30 AM, multiple states emerge for percolation under a given congestion rate, as seen in Fig. 2B. This multistate behavior is validated statistically (SI Appendix, Figs. S3-1–S3-4). Interestingly, these two behaviors overlap in the upper-state regime in the range of $f \in [0, 0.2]$, yet during 7:30 to 8:30 AM, there also exists a lower-state regime for Gwith $f \in [0.1, 0.3]$. Thus, while in the period during 6:00 to 6:30 AM one state exists, and during 7:30 to 8:30 AM two states exist. We also observe similar (but weaker) multistate behavior during evening rush hours (SI Appendix, Figs. S1-6 and S1-7). However, two such states almost do not appear in days off (*SI Appendix*, Fig. S1-8).

To infer the shape of the attraction basin of each regime, we calculate the size distribution of the largest functional cluster (G), for different f values. As seen in Fig. 2C, for a low congestion rate f (e.g., f = 0.1), urban traffic is unimodal with a peak at high performance $(G \approx 0.9)$; when f increases, more roads become congested and the probability of being in the high-performance regime decreases, leading to a transition to a low-performance regime. Two alternative regimes can be seen to coexist for a wide range of values of the congestion rate, f. In this way, the two possible states for the same f in Fig. 2A can be separated by a suitable boundary line (details can be found in *Methods*). For a congestion rate f greater than about 0.4, there exists only one state, namely the low-functioning state.

To visualize the distribution of states over time, we combined results from different days into a single plot, as seen in Fig. 3A. Two critical points in time can be observed. At the first point around 7:00 AM, the L state begins to appear, and the system can exist in either the L state or the H state. The competition between the two states becomes stronger as time progresses. This competition ends at the second critical point around 9:00 AM, when the system shows only the high-performance state. These two instants could help predict the likelihood of traffic operational state and suggest possible real-time resilience management schemes for different congestion stages. We also find in Fig. 3B that the occurrence of the two alternating states is slightly different on different working days, which represents overall traffic performance for each day. Meanwhile, on October 21, the system stays mostly in the low-performance regime, while on October 23, the fraction of the H state is much more significant than that of the L state. Note the fact that October 21 is a traditional festival in China in the lunar calendar, although it is still officially a workday. The system state can show a "temporal persistence," as seen in Fig. 3C. When considering autocorrelation

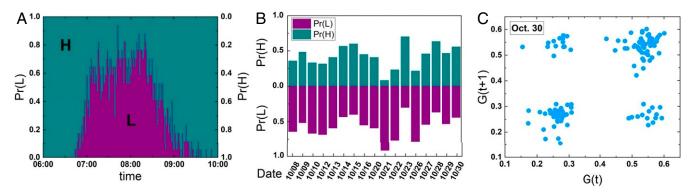


Fig. 3. Evolution of alternating states in a traffic network. (*A*) Probability of different system states as a function of time under a given congestion rate f = 0.25 in 17 workdays. (*B*) Probability of the system state during 7:00 to 9:00 AM in every workday under a given congestion rate f = 0.25. (C) Autocorrelation (i.e., G(t + 1) versus G(t)) for morning hours during 7:00 to 9:00 AM in a typical day (October 30), given f = 0.25.

of the system performance (G) for a given day, it is clearly seen that the high-high transition and low-low transition appear more frequently for a given working day compared to high-low and low-high switches (more daily results can be seen in *SI Appendix*, Fig. S4-4).

To avoid system collapse, it is critical to locate the tipping point of the system through abrupt shifting to the lowperformance regime. As shown in Fig. 2*A*, we observe two critical fractions, within which the multiple system states exist. These two congestion rates indicate two tipping points which would require the adoption of different traffic control strategies. Previous works in other fields (15–18) have proposed many methods to locate the tipping points of multistate systems through various temporal signals. However, it is difficult to apply these methods here due to the noise and instability of traffic dynamics. Therefore, as seen in Fig. 4 A and B, we show how we apply our percolation method to locate these tipping points. According to the theory of phase transitions in percolation, much can be learned from studying the scaling behavior when approaching the critical point. For example, when the system is close to the percolation critical point, f_c , the order parameter of the system, i.e., the giant component size G, varies as (19)

$$|G - G_c| \sim |f - f_c|^{\beta}.$$
 [1]

Here f_c is the critical point where a system phase transition can occur, and the critical exponent β tells us how fast the order parameter vanishes at the critical point. We locate simultaneously both the critical point f_c and the critical exponent β by calculating, for each day, the scaling behavior of G in the

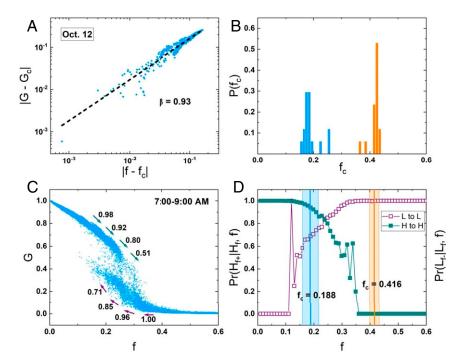


Fig. 4. Criticality and persistence of urban traffic system. (A) The scaling relationship between $|G - G_c|$ and $|f - f_c|$, where f_c (around 0.16 in this case) is determined by the (best) fit in a log-log plot with the largest statistic R^2 (details can be seen in *SI Appendix*, Fig. S5). (B) Distribution of f_c for each workday obtained by the critical scaling method. It is found that the values of f_c are concentrated in two narrow ranges. (C) Schematic demonstration of the persistence of the traffic network state. With adiabatic changes of congestion rate f, the system is more likely to persist in the same state as before. (D) System state persistence during 7:00 to 9:00 AM in every workday. H_f or L_f is the system state when the congestion rate is f, whereas H_{f+} or L_{f-} is the system state when the congestion rate increases or decreases slightly (by $\Delta f = 0.01$). The mean values of f_c obtained in B, with SD, are also shown here.

vicinity of the critical point (G_c represents the largest functional cluster size at f_c). Note that here we analyze $|G - G_c|$ as a function of $|f - f_c|$ instead of simply G, considering the transition in this dynamical percolation is likely a hybrid one (20), as shown in Fig. 4A. More results for specific days can be found in *SI Appendix*, Fig. S5. The two critical points are similar for different days, as seen in Fig. 4B. We also find that the critical exponent β is very similar and thus assumed to be universal across all days, suggesting that it relates to fundamental symmetries in the traffic percolation of Beijing (*SI Appendix*, Fig. S5).

The observation of metastability seen in Fig. 2A suggests a possible hysteresis-like behavior. One fundamental feature of hysteresis is persistence, where the system prefers to remain in the same state in the hysteresis regime. Following this idea, we study and test the persistence of each state. Focusing on the time during 7:00 to 9:00 AM in all of the working days, we observe significant persistence in the traffic percolation. That is, with adiabatic changes (i.e., small increases or decreases) of the congestion rate f, the system is more likely to persist in its previous state, as seen in Fig. 4 C and D. Statistically, we define the conditional transition probability $Pr(H_{f+}|H_f, f)$, where H_f means the system being in the H state for the current congestion rate f, while H_{f+} refers to the system remaining in the H state if the congestion rate increases by Δf (here we set $\Delta f = 0.01$). A similar definition can be made for the system's likelihood of staying in the L state as the current congestion rate f is decreased and is indicated by $Pr(L_{f-}|L_f, f)$. As seen in Fig. 4D, we find that the traffic network is more likely to "memorize" its previous state within the metastable region. The existence of this persistence in traffic represents a signature of metastability and hysteresis at the entire network scale (21).

Discussion

Urban traffic, as a nonequilibrium complex system, undergoes in megacities critical transitions almost every day. Faced with the challenge of increasing uncertainty, and various perturbations from extreme climate to collision accidents, it is essential to understand and explore the nature of the system's adaptation and recovery. Here, we identified multiple states in functional traffic networks under the same perturbation at the scale of the entire city. These phenomena may be due to the existence of long-range correlations. Based on real data, the emergence of long-range correlations in traffic congestion has been found (22) during the rush hours. Indeed, it has also been suggested (23) that long-range connections can determine the overall flow conductance of the network. This long-range correlation, combined with the dynamical fluctuations of commute traffic in the megacity, may generate the multiple macroscopic states, which can be identified by our percolation method. Moreover, we propose a statistical approach to accurately locate the critical point separating the region where a single state exists and the region of multiple states. We find that the multiple states of the system and the transitions between them are highly parallel to the hysteresis phenomenon from statistical physics. Precisely, we observe that the system experiences strong persistence in the metastable regime. For adiabatic changes (small changes) in f, the system's state is highly dependent on its previous state with the system being more likely to remain in the same state. Nonetheless, we refer to this phenomenon as "hysteresis-like" since the mechanisms behind this pattern require further study in the future. Even still, the parallels are remarkable and strongly justify the applications of statistical physics to complex systems such as urban traffic.

There are several future implications based on our current results. The first implication is the critical point identified in our study. With this critical point, we can design early warning signals based on real data of traffic velocity or flux. When this signal is measured showing the traffic system is approaching the critical transition, one could implement more specified traffic signal controls (24) or congestion toll (25) onto bottleneck areas to adjust the traffic flow dynamically. Furthermore, if we enter the multistate regime of system dynamics, certain mitigation methods could be developed in the future to help the system to maintain the high-performance state or to recover soon from the low-performance state. The above implications could be realized with intelligent transportation technologies through coordinating road states, car behaviors, and human decisions, with smart agents deployed across different levels. In contrast to most proposed methods based on car density control, our findings suggest an insightful direction for traffic control. Namely, to avoid the abrupt regime shift from a high-performance regime to a low-performance one, we should focus on decreasing the number of congested roads before the network reaches its tipping point.

These multiple states may also explain the origin of high travel heterogeneity from the individual's perspective (26). In the highperformance state, a driver will enter the largest cluster of good traffic with a high probability and feel a smooth driving experience. However, in the low-performance state, the driver will feel a very fragmented and fluctuating traffic experience. Normally, we expect a similar traffic experience along the same commute route every morning. However, if we compare the driving experience on the days of October 21 and October 23 with different state configurations, one will feel contradicting traffic experiences at the same time in these different days.

Given Holling's (4) definition of resilience in ecology, resilience studies in engineering fields have recently begun to focus on the system's ability to absorb, adapt, and recover (27, 28). While traditional system reliability engineering has focused on the design of a system to avoid operational collapse, the more recent study of system resilience has shifted the viewpoint for system reliability and safety from "safe-fail" to "fail-safe" (29), suggesting a systematic management paradigm to recover from unexpected disturbances. It is found that many natural systems can become stabilized at different state levels under the influence of random events, which suggests the existence of different domains of attraction in these systems (4). In this sense, resilience suggests multiple metastable states in ecology, climate, and biology systems. Meanwhile, for engineering systems including critical infrastructures, it is usually assumed that the system has only one single equilibrium state and will return to this original state after perturbations. Identification of multiple network states suggests the necessity of paradigm shift for the corresponding complex system management.

Our methods consider both network structure and dynamics, which can be easily generalized to many other complex networks. By learning the spatiotemporal features of system transitions, our resilience theory can help develop sound predictive methods. These can then be applied to a wide range of fields including avoiding epileptic seizures in the brain (5), predicting extreme weather in climate systems (6), foreseeing epidemic outbreaks (30), and avoiding collapse in financial systems (31). Based on the concept of critical slowing down (32), many temporal indicators have also been proposed including autocorrelation and variance. However, these temporal indicators usually require high-resolution data of long time series and become less reliable when the level of noise is high (8). Changing external perturbations on the supply and periodic demand in critical infrastructures makes it hard to predict their closeness to a tipping point using traditional temporal indicators. In some practical applications, spatial indicators are instead considered to provide more information and possible early warnings at short time scales. For complex systems, it remains an urgent challenge to identify suitable indicators for the system collapse. Our percolation methods may provide a possible direction toward achieving that goal.

Theoretical understanding for multiple metastable states, especially for network scientists, is very interesting yet challenging. How the network structure could generate the dynamics of multiple metastable states is a valuable question. Dynamical models of synchronization (33) or infectious process (21) are found to exhibit multiple states in networks. For realistic traffic dynamics with more complexity, the underlying network is embedded into spatial space, where the geometry of network structure may play a critical role. Therefore, further studies are needed for unveiling the relationship between system structure and metastability.

Methods

Traffic Percolation. We study the percolation of city traffic as follows. At a given time, we set a threshold q from 0 to 1 with a given interval (e.g., 0.1 or 0.05). We compare the relative velocity of a road for a given q: If the relative velocity is larger than q, then we consider the road functional at this given time; otherwise it is considered failed and removed from the original road network. By calculating the fraction of failed roads with respect to the total number of roads in the entire road network for a given q, we can obtain correspondingly the congestion rate, represented by f. We use the size of the largest functional cluster (denoted by G) to represent the state of the overall traffic at the scale of the whole network. Large values of G represent a larger connected functional network.

Resilience Function. For a given removal fraction f of congested roads, we analyze the distribution of the largest functional cluster size, G, for all of the real-time cases covering 17 working days. Although the statistical validation can identify more than two states for a given f, we find that these states can be clustered into two main regimes. For instance, in Fig. 2C when f = 0.24, two macroscopic states exist with two maxima corresponding to approximately $G \approx 0.3$ and $G \approx 0.6$. Therefore, we can find the two maxima values in the distribution of G for a given f and make a plot of the corresponding maximal G versus f. Using a polynomial fit, we can obtain the optimal resilience function for each regime. We can also determine the separation boundary line between the two regimes by fitting the minima of the state distributions. Our percolation methods should be combined with statistical methods in the future to identify the attraction basin accurately. Given the scattered values within f ranging between 0.1 and 0.4, if a state is above the separation boundary line (dashed line in Fig. 2A), we regard it as an H state; otherwise we define it as an L state.

Overall, we assume the boundary line has the form of

$$G_{s}(f) = B_{2} \cdot f^{2} + B_{1} \cdot f + B_{0}, f \in [0.1, 0.4].$$

 B_2 , B_1 , and B_0 are the parameters derived from fitting. For the system performance represented by G (f) at a given congestion rate f, if f < 0.1, then the system is classified in the H-state regime; if f > 0.4, then it is in the L-state regime. Within the range $0.1 \le f \le 0.4$, we can compare the value of G (f) and G_s (f). If $G(f) \ge G_s(f)$, then the system is in the H-state regime; otherwise it is in the L-state regime. Namely, within the above metastable range, we use the following criteria:

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$$S(G(f), f, 0.1 \le f \le 0.4) = \begin{cases} H, G(f) \ge G_s(f) \\ L, G(f) < G_s(f). \end{cases}$$

At each instant for a given f, the system state can be either an H state or an L state. Therefore, we can calculate the frequency of the H state or the L state for each day, indicated as Pr(H) or Pr(L) in Fig. 3B.

State Persistence. When dealing with system state persistence, one should perform adiabatic changes (small changes) of the control parameter, to test whether the system's state is highly dependent on its previous state (i.e., the system is more likely to remain in the same state). By this means, a possible hysteresis region can form (as shown in Fig. 4C). Therefore, we use the following formula to calculate the conditional probability of how the system state will evolve (i.e., persist in or change its current state) under a given condition, including the current congestion rate (f) and current system state (H or L). This conditional probability can be indicated by $Pr(H_{f+}|H_f, f)$ and $Pr(L_{f-}|L_f, f)$:

$$\begin{split} & \Pr(H_{f+} \mid H_f, f) \stackrel{\text{def}}{=} \begin{cases} \frac{\Pr(H_{f+}, H_f, f)}{\Pr(H_f, f)}, & \Pr(H_f, f) > 0\\ 0, & \Pr(H_f, f) = 0, \end{cases} \\ & \Pr(L_{f-} \mid L_f, f) \stackrel{\text{def}}{=} \begin{cases} \frac{\Pr(L_{f-}, L_f, f)}{\Pr(L_f, f)}, & \Pr(L_f, f) > 0\\ 0, & \Pr(L_f, f) = 0. \end{cases} \end{split}$$

Here H_f or L_f is the system's state (H or L state, respectively) for the current congestion rate f, while H_{f+} or L_{f-} is the system state when the congestion rate f changes slightly (increasingly for "+" while decreasingly for "-," by $\Delta f = 0.01$) based on the current condition. $\Pr(H_f, f)$ or $\Pr(L_f, f)$ is the probability that system state is an H or L state for a given congestion rate f. $\Pr(H_{f+}, H_f, f)$ stands for the probability that system persists in the H-state regime with increasing f at that same instant. Similarly, $\Pr(L_{f-}, L_f, f)$ is the probability that same instant. Note that if $\Pr(H_{f+}, f)$ or $\Pr(L_f, f)$ is equal to 0, we set $\Pr(H_{f+} | H_f, f)$ or $\Pr(L_{f-} | L_f, f)$ as 0.

Data Availability. The datasets analyzed in the current study are not publicly available under the restrictions of the data provider, but they are available upon request from the corresponding author(s).

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