International Journal of Disaster Risk Reduction 40 (2019) 101266

Contents lists available at ScienceDirect

International Journal of Disaster Risk Reduction

journal homepage: www.elsevier.com/locate/ijdrr

# Recent progress on cascading failures and recovery in interdependent networks

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ARTICLE INFO

*Keywords:*
Network of networks (NON)
Cascading failures
Degree-correlations
Intersimilarity
Modularity structure
Spatially-embedded networks
Spontaneous recovery
Protection

ABSTRACT

Complex networks have gained much attention in the past 20 years, with thousands of publications due to their broad interest and applicability. Studies initially focused on the functionality of isolated single networks. However, crucial communication systems, infrastructure networks and others are usually coupled together and can be modeled as interdependent networks, hence, since 2010 the focus has shifted to the study of the more general and realistic case of coupled networks, called Networks of Networks (NON). Due to interdependencies between the networks, NON can suffer from cascading failures leading to abrupt catastrophic collapse. In this review, using the perspective of statistical physics and network science, we will mainly discuss recent progress in understanding the robustness of NON having cascading failures features that are realistic for infrastructure networks. We also discuss in this review strategies of protecting and repairing NON.

## 1. Introduction

Modern infrastructure systems are rarely isolated, but rather are usually coupled together through complex interdependencies [1–4], such as the relationship between water and food supply, communications, fuel, financial transactions, power stations and others (see Fig. 1). The coupling between different systems can lead to cascading failures and catastrophic consequences in the entire network of networks (NON) system. This is since nodes that fail in one layer will cause dependent nodes in other layers to also fail, which in turn may cause further failures back and forth from one infrastructure to the others. To understand the cascading failure effect of this interdependence on the structural and functional behavior of the entire infrastructure system, a framework based on percolation theory has been developed [1–8]. This framework studies the dynamics of cascading failures and predicts critical points of failure transitions. Many researchers have investigated this system assuming random-couplings, degree-correlations [9–12], inter-similarity [13–18], modularity structure [19,20], underlying spatial structures [21–23], protecting and repairing strategies [24–27], and discovered rich and surprising behaviors in NON.

By generalizing the percolation theory of a single network to the case of NON [1,3,5], the results show that the former is only a specific limited case of the latter. In the original work [1], Buldyrev et al. developed an analytical framework for two fully interdependent networks by advanced techniques from statistical physics and found that the system undergoes an abrupt transition. Others expanded this work by considering partial coupling [3], more than two networks [5,28,29], various correlations including degree-correlation [2,11,12,17,30–33] and intersimilarity [14–16,18], which led to improved understanding of cascading failures in more realistic NON. Recently, to adequately model real-world systems, researchers have also begun to study the robustness of real networks with community structures [19,20,34] and developed efficient strategies of protecting and repairing NON [22,23,35]. This is since avoiding such a dramatic breakdown and reducing the fragility of real systems is of critical importance.

In this review, we illustrate the framework for percolation on early models of interdependent networks in Section 2, and review the phenomena of cascading failures found in realistic models of interdependent network structures in Section 3. In Section 4 we discuss the extreme vulnerability of interdependent spatially-embedded networks against random attacks and in particular localized attacks. Finally, we summarize some efficient protection strategies in such systems in Section 5.

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https://doi.org/10.1016/j.ijdrr.2019.101266
Received 5 June 2019; Accepted 4 August 2019
Available online 07 August 2019
2212-4209/ 

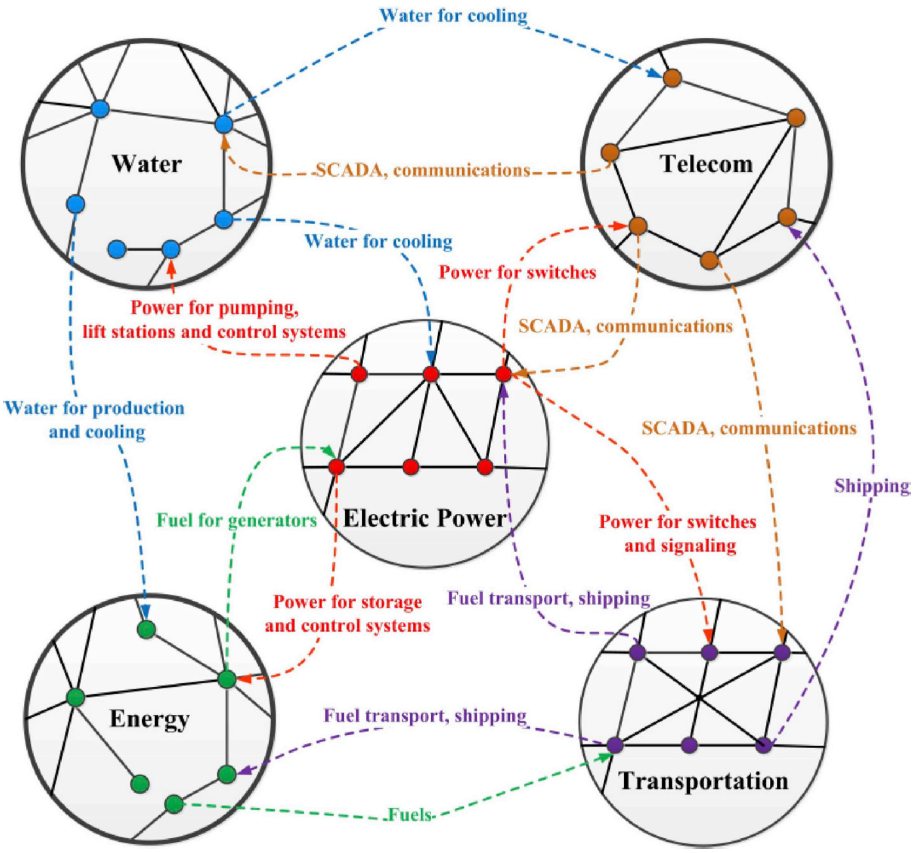


**Fig. 1.** (Color online) Illustration of the interdependent relationship among different infrastructures [37]. After Ref. [38].

## 2. Early models of interdependent networks

Some original works [1,3] found that interdependence between networks can highly increase their vulnerability under random attacks compared to single networks. For instance, an initial failure of only one power station may lead to an iterative cascade of failures that causes the interdependent system of the power grid and the communication network to become fragmented (see demonstration in Fig. 2). Such phenomenon was observed in the Italian Blackout of 2003 [36] and it is a great challenge to protect such infrastructure networks. Therefore, mathematical methods are needed to understand how interdependence between networks affects the system's robustness.

### 2.1. Percolation in two-layer networks with full interdependence

Buldyrev et al. [1] considered two randomly connected networks with full interdependence (see Fig. 3), and developed a mathematical framework to discover that the cascading failures lead to an abrupt

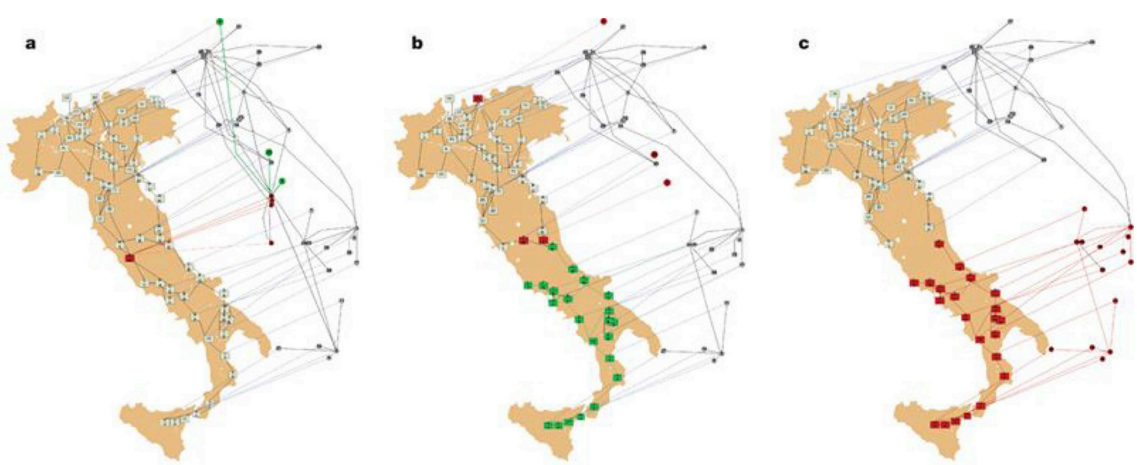


**Fig. 2.** Illustration of an iterative process of a cascade of failures using a real-world example from the power network (located in Italy) and communications network (shifted above the map) that were implicated in the electrical blackout that occurred in Italy in September 2003. Both the removed nodes and their dependent nodes in the other layer are marked in red, the nodes that will be disconnected from the giant component at the next step are marked in green. **a** One power station that failed in the power network caused the dependent nodes in communication systems to also fail. Additional nodes that will fail in the communications network are marked in green (above the map). **b** The nodes in both networks that will fail in the next iteration are marked in red. **c** Further nodes that fail in both the power grid and corresponding dependent nodes in the communications network are marked in red. After Ref. [1].

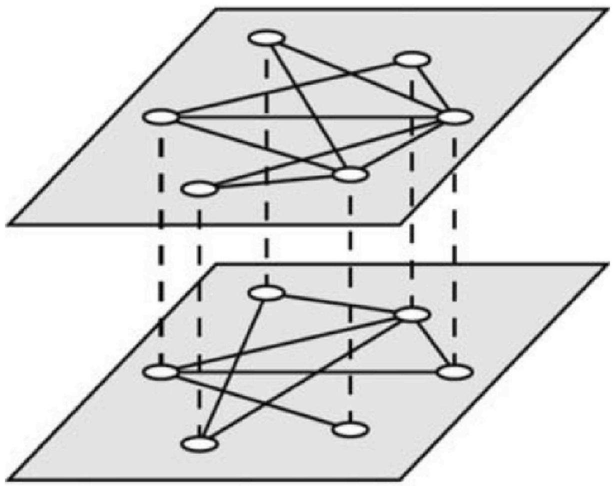

**Fig. 3.** Two randomly connected networks with full interdependence. After Ref. [40].

phase transition. In their model, each node in network A depends on a node in network B and vice versa. They assumed two layers each with the same number of nodes N, with degree distributions $P_A(k)$ and $P_B(k)$, respectively. After initially removing a fraction $1-p$ of nodes (and their associated edges) from network A, they carry out an iterative process of cascading failures according to the interdependence between the networks. We let $f_A(p)$ (or $f_B(p)$) be the probability that a randomly selected link in network A (or network B) is not in the mutual giant component, and introduce the generating functions of the degree distribution and the underlying branching process as $G_{A0}(z)=\sum_k P_A(k)z^k$ and $G_{A1}(z)=\sum_k kp_A(k)z^{k-1}$ respectively. Thus, the probability that a randomly selected node belongs to the giant component after the removal of $1-p$ fraction of nodes is

$$g_A(p)=1-G_{A0}(1-p+pf_A(p)),\ \ f_A(p)=G_{A1}(1-p+pf_A(p)). \tag{1}$$

To determine the size of the giant mutually connected component $p_\infty$, we define x as the probability that a randomly chosen node in network A remains functional after the iterative cascade process. Analogously, we define y as the probability that a randomly chosen node in network B remains functional. Thus

$$x=pg_B(y),\ \ y=pg_A(x),\ \ p_\infty=xg_A(x),\ \text{or}\ p_\infty=yg_B(y), \tag{2}$$

where $g_A(y)$ are defined by Eq. (1), and $g_B(x)$ is defined similar to $g_A(y)$. Eq. (2) is identical to Eqs. (12)–(14) in Ref. [4], after substituting $(1-f_A)x$ and $(1-f_B)y$ for $x$ and $y$. At the critical point of transition, $p=p_c$, the two functions $x=pg_B(y)$, $y=pg_A(x)$ are tangent with each other (see also [5,29,39]),

$$\frac{d(pg_A(y))}{dy}\frac{d(pg_B(x))}{dx}=1. \tag{3}$$

Using the above theory, Ref. [1] discovered that the cascading failures yield an abrupt, discontinuous phase transition in coupled Erdős-Rényi networks (ERs). This is dramatically different from the continuous phase transition due to failed nodes found in isolated ER networks (Fig. 4a). For two randomly coupled scale-free networks (with $P_A(k)=P_B(k)\propto k^{-\gamma}$, $\gamma\le 3$), phase transition takes place at a nonzero critical value $p_c\neq 0$, even for $2<\gamma\le 3$, in contrast to the case of $p_c=0$ in an isolated single network [41]. Furthermore, analytic predictions and simulation results showed that a broader degree distribution would increase the vulnerability of interdependent networks to random failure compared to their non-interacting counterparts. All these results illustrate that interdependence can highly increase the vulnerability of an infrastructure system, since the failure of a randomly selected node in one network may cause a failure of a hub in a second network, which in turn triggers a catastrophic collapse of the system [1,3].

### 2.2. Cascading failures in two-layer networks with partial interdependence

Generalizing the corresponding theory of Ref. [1], Parshani et al. [3] proposed a system composed of two layers (A and B) with partial interdependence, and found that as the coupling strength decreases, there is a critical point where the phase transition switched from abrupt to continuous. Specifically, a partially interdependent network is similar to the previous model [1], but now only a fraction $q_A$ of nodes in layer A depend on nodes in layer B and a fraction $q_B$ of nodes in layer B depend on nodes in layer A. Thus the corresponding analytical framework becomes,

$$\begin{aligned} x&=p\{1-q_A[1-g_B(y)]\},\\ y&=1-q_B[1-pg_A(x)],\\ p_\infty&=xg_A(x),\ \text{or}\ p_\infty=yg_B(y), \end{aligned} \tag{4}$$

which is similar to Eq. (2), and the variables p and $p_\infty$ are the same as in Ref. [1]. Using Eq. (4) and the requirement that at criticality the top two functions in Eq. (4) meeting tangentially with each other, Parshani et al. obtained analytical and numerical results for percolation in partially coupled ER networks (see Fig. 3 in Ref. [3]). That is, reducing the coupling strength will change the type of the percolation-phase transition, since for strong coupling (large values of $q_A$ and $q_B$) the system undergoes a first order transition, while for weak coupling it undergoes a second order phase transition (Fig. 4b).

### 2.3. Cascading failures in n-layer networks with full or partial interdependence

The case of n-layer interdependent networks [5,28,29,39,42,43] is a generalization of the two-layer case. Assume that each of n interdependent networks consist of the same number of nodes, and each node in one layer is mutually interdependent on only one node in other interdependent layers via one-to-one matching. We review here, results of percolation on n interdependent networks through four examples which can be analytically solved. (I) a tree-like network of n ER networks with full interdependence [28,29], (II) a loop like network of ER networks with partial interdependence [5,28,29], (III) a random regular network composed of partially interdependent ER networks [29,39] (in which each ER network is dependent on exactly m other ER networks), and (IV) n scale free (SF) networks with partial interdependence [43]. All cases represent different generalizations of percolation theory in a single network.

For case (I), we assume each layer has the same average degree $\langle k\rangle$, Gao et al. [28] showed that the tree-like structure with full interdependence becomes more vulnerable with increasing n or with decreasing average degree $\langle k\rangle$. Moreover, the final size of the mutual giant component follows

$$p_\infty=p\left(1-e^{-\langle k\rangle p_\infty}\right)^n. \tag{5}$$

For n = 1, Eq. (5) reduces to the known result of a single ER network [44], and the system shows a continuous transition, while for any $n>1$, the system undergoes an abrupt phase transition (see Fig. 5a). In the following, assume that each partially dependent pair of networks has the same fraction of dependency nodes q. For the interdependent networks of models (II) and (III), it is very interesting that both the critical threshold and the giant component do not depend on the number of networks n, in contrast to the tree-like structure, they only depend on the coupling strength q and m but not on n (Fig. 5b and c). Especially for case (III), the giant component of $p_\infty$ can be solved analytically

$$p_\infty=\tfrac{p}{2^m}(1-e^{-\langle k\rangle p_\infty})(1-q+\sqrt{1-q^2+4qp_\infty})^m. \tag{6}$$

Interestingly, Ref. [39] found a critical coupling strength $q_c$ that distinguishes between first and second order transitions,

$$q_c=\frac{\langle k\rangle+m-(m^2+2\langle k\rangle m)^{\frac{1}{2}}}{\langle k\rangle}. \tag{7}$$

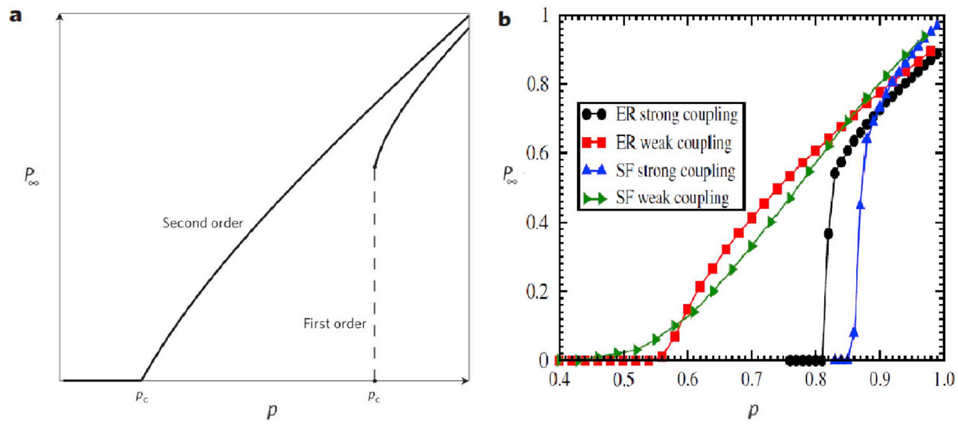


**Fig. 4.** (Color online) **a**, Schematic demonstration of first-(abrupt) and second-order (continuous) percolation transitions. **b**, For strong coupling strength, $p_\infty$ jumps abruptly [ER (circle) and SF (up-pointing triangle)]. For weak coupling strength, $p_\infty$ changes continuously to 0 [ER (square) and SF (right pointing triangle)]. After Ref. [3].

For $q < q_c$ the percolation transition is continuous (second order), while for $q > q_c$ it is abrupt (first order). In case (IV), each network has the same degree distribution $p(k) \propto k^{-\gamma}$. For n networks that are randomly connected, there exist two critical coupling strengths $q_c$ and $q_{max}$ [39]. That is, for $q < q_c$, the system undergoes a continuous phase transition, while for $q_c < q < q_{max}$, the system undergoes an abrupt transition [43,45]. Surprisingly, when the coupling strength q is greater than $q_{max}$, the entire system will collapse even if only one node fails, that is, in this case $p_c = 1$.

## 3. Realistic interdependent network structures

To adequately model most real-world systems, we must recognize that both the dependency and connectivity links are not random, but rather have structural patterns such as degree correlation [11,12,30–32] and intersimilar links [14–16,18]. These correlations can dramatically affect the dynamical properties and structural robustness of the interdependent networks [46]. Moreover, networks in nature also have internal community structures, related to groupings into cities and other geographical regions [19,20]. Thus, here we review some recent process on how these realistic interdependent network structures affect the robustness of the entire system.

### *3.1. Interlayer degree correlations in interdependent networks*

Here we will focus on patterns of interdependence between nodes in the different layers. We will assume that there exists in each layer an ordering of nodes from highest degree to lowest degree. We will consider three cases of assigning dependency links between nodes in the different layers based on the correlation of the node's degrees. The cases are: maximally positive (MP), maximally negative (MN), and uncorrelated (UC) multiplex structures, following Ref. [11]. Based on the multivariate generating function method, Ref. [11] addressed various robustness properties of interdependent networks composed of two ER networks, such as the resilience of ordinary and mutual connectivity under random or targeted node removals. The analytic prediction, together with numerical simulations, shows that the correlated coupling of multiplex layers plays an important role in changing the robustness of the multiplex networks, i.e., under random node failures, higher correlations (MP case) in degree between interdependent nodes leads to a lower percolation threshold, whereas a higher percolation threshold is obtained in anticorrelated interdependent networks (MN case). For targeted attack based on node degrees, on the contrary, positively correlated interdependent networks (MP case) are highly vulnerable, but the anticorrelated networks (MN case) can become more robust.

### *3.2. Interdependent networks with intra-degree and inter-degree correlations*

Reis et al. [12] showed that the stability of a system of interdependent networks not only depends on the intra-structure of each layer, but also on the inter-structure between layers. They consider two correlated SFs, setting k as intra-links degree of the nodes, and denoting $k_{out}$ as the degree of the inter-links, which are characterized as $k_{out} \sim k^{\alpha}$. For $\alpha > 0$, the hubs in one network will typically have more inter-links, whereas for $\alpha < 0$, the inter-links of a hub node are more likely to be

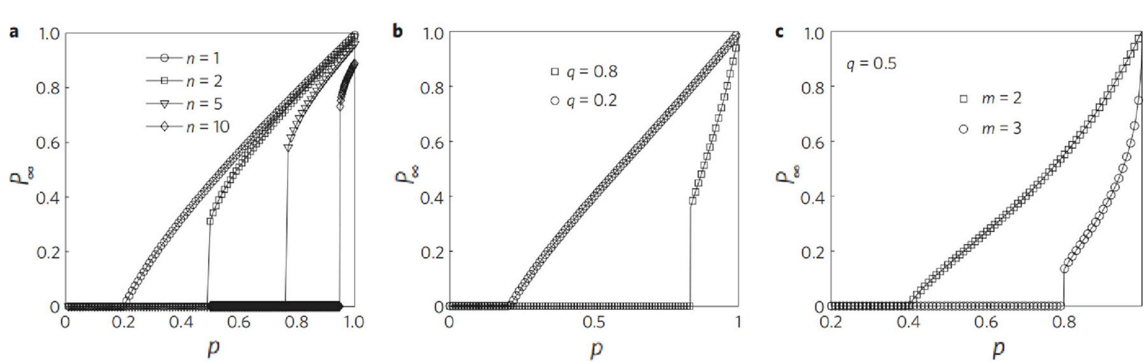


**Fig. 5. a**, A tree-like system of n interdependent ER networks. We show $p_\infty$ versus p for $\langle k \rangle = 5$ and several values of n. Increasing n ($n \geq 2$) yields a first-order transition. **b**, A loop like network of interdependent ER networks with partial interdependence. We show $p_\infty$ versus p for $\langle k \rangle = 6$ and two values of q. Increasing q yields a first-order transition. **c**, An n-layer network in which each ER network is dependent on exactly m other ER networks. We show $p_\infty$ versus p for $q = 0.5$. Here, changing m from 2 to $m > 2$ changes the transition from second order to first order. After Ref. [29].

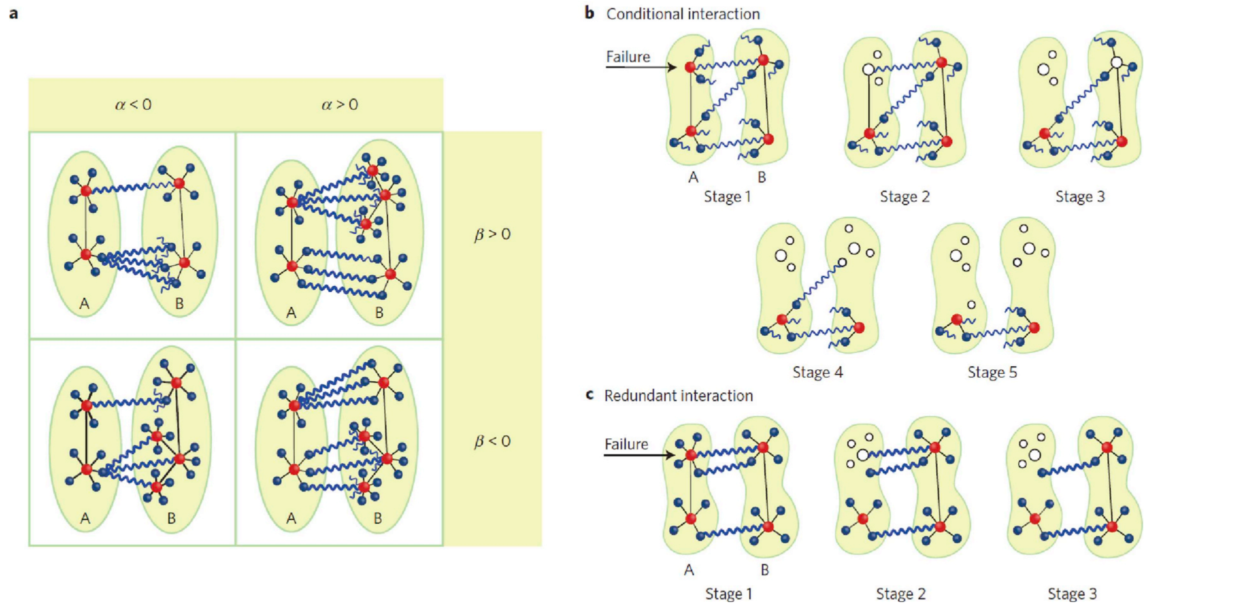


**Fig. 6. a**, Hubs (red nodes) and non-hubs (blue nodes) have intra-links (solid black links) and inter-links (wiggly blue links) according to the parameters α and β. **b**, Conditional mode of failure: a node fails every time when it becomes disconnected from the largest component of its own network, or loses all its outgoing links. **c**, Redundant interaction: the failure of a node only leads to further failure if its removal isolates its neighbors in the same network. After Ref. [12].

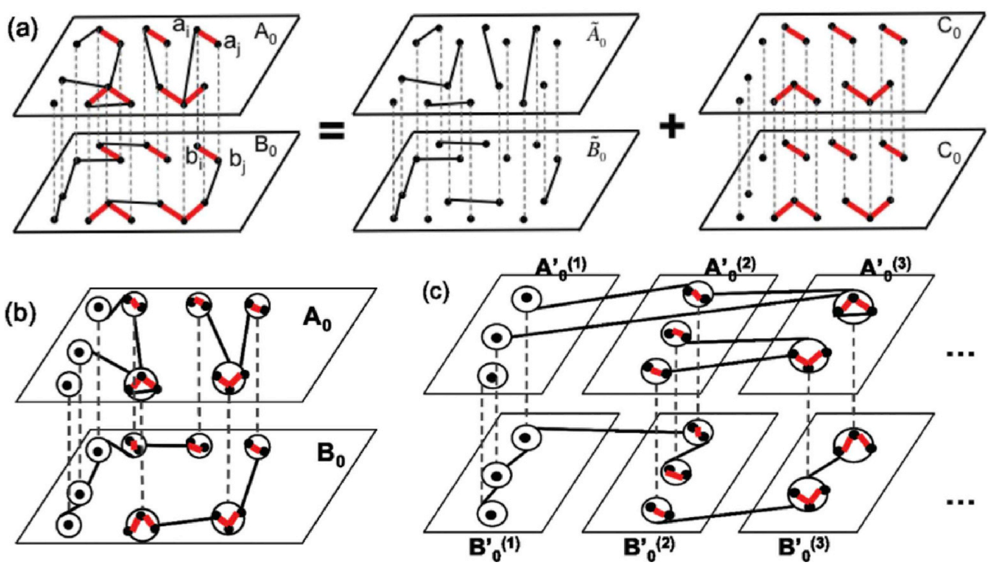


**Fig. 7.** (Color online) The remaining network after initial attack. **(a)** Interdependent networks $A_0$ and $B_0$ with overlapped links. Interdependent nodes are connected by dashed lines. $A_0$ and $B_0$ can be decomposed into three subnetworks $\widetilde{A_0}$, $\widetilde{B_0}$ and $C_0$. Black links in $\widetilde{A_0}$ and $\widetilde{B_0}$ are not overlapped. Red thick lines in $C_0$ are all overlapped links. **(b)** Contracted network with supernode. Since nodes in the same component of $C_0$ will survive or fail simultaneously, a component in $C_0$ can be replaced by a supernode. **(c)** The contracted network is further decomposed into subnetworks according to the size of the component. For instance, nodes in ${A_0}'^{(3)}$ are supernodes converted from a component of size 3 in $C_0$. After Ref. [14].

less (Fig. 6a). Nodes of different networks are connected according to $\beta$, $k^{nn}{\sim}k^{\beta}$, where $k^{nn}$ is the average intra-degree of the nearest neighbors of a node in the other network. When $\beta > 0$, nodes with similar degree prefer to connect between themselves, and when $\beta < 0$, nodes connect disassortatively (Fig. 6a). If $\alpha = 0$ and $\beta = 0$, the interdependent network is uncorrelated.

Reis et al. [12] analyzes the cascading failure of the interdependent networks in two different manners. One is conditional interaction, a node in a given layer fails every time when it becomes disconnected from the largest component of its own network, or loses all its interlinks (Fig. 6b). The other manner is the redundant interaction, i.e., the failure of a node only leads to further failure if its removal isolates its neighbors in the same layer. Thus a node may survive even if it is completely decoupled from the other layer, as long as it is still remains attached to the largest component of its own network (Fig. 6c). In the conditional manner, the interdependent network is stable for $\alpha < 0$, where the hubs are protected from possessing more inter-layer links, and stable for $\alpha \gtrsim 0.5$ and $\beta > 0$, where the inter-layer connections are convergent to some extent. Instead, it becomes particularly unstable for intermediate values of $0 < \alpha < 0.5$ and $\beta < 0$, since hubs interdependent with low-degree nodes ($\beta < 0$) can be easily attacked via conditional interactions, and lead to catastrophic cascading after the attack. In the redundant interaction, the interdependent network is more unstable for $\alpha < 0$, where the hubs have less dependency links. Correspondingly, the large non-hub nodes have more interdependent links, their failures and their interdependent links' failures will cause the failure of the hubs in the other layer, and vice versa. However, the system becomes stable for $\alpha \approx 1$ regardless of $\beta$. Here the more interdependent links the hubs have, the more easy it is to cripple the function of the hubs in the other layer.

### 3.3. Percolation on interdependent network with edge correlation

In this section, we review studies of models of percolation in interdependent networks with edge correlation. Edge correlation can be measured by intersimilarity. Intersimilarity represents the ratio of overlapped edges between interdependent networks. Notice that, overlapped edges (see Fig. 7) imply a pair of interdependent nodes whose neighbors in both networks are also interdependent. For example, a cargo port network and an airport network could have high intersimilarity, if there are many direct flights and direct shipping lines to the same cities. To understand the effects of intersimilarity on the cascading failures of a system, Hu et al. [14], Wang et al. [15], Min et al. [16], Parshani et al. [17] and Baxter et al. [18] demonstrated that high intersimilarity increases the robustness and can change the order of the phase transition in interdependent networks from first order (abrupt) to second order (continuous).

#### 3.3.1. Percolation of interdependent networks with intersimilarity

To quantify the intersimilarity, we decompose the remaining networks ($A_0$ and $B_0$) after initial attack into two interdependent subnetworks consisting of all the nodes of $A_0$ and $B_0$. One of them has all the overlapping links, the other has all the nonoverlapping links (see Fig. 7). Hu et al. [14] found that if a node in a component of $C_0$ survives in a cascading failure, the whole component will survive and vice versa. This property changes the process of the failure. They find that if interdependent networks are fully intersimilar or identical (all links overlapping), the percolation is identical as percolation on a single network. However, if the interdependent networks are only partially intersimilar, the transition is always first-order rather than second-order.

#### 3.3.2. Group percolation in interdependent networks

Wang et al. [15] proposed a general model of group percolation. In this model, nodes in fully interdependent networks are divided into n groups respectively, and each group is replaced by a supernode. Two supernodes are connected by a link if and only if there are links between regular nodes from the two corresponding groups. Every supernode depends on one and only one supernode in the other network. They proved that continuous phase transitions do not exist in either original networks nor the contracted networks. Moreover, the previous model in Ref. [14] can be considered as a special case within the framework of group percolation in Fig. 8. This reiterates the conclusions in Ref. [14] and shows that intersimilarity enhances the system robustness.

#### 3.3.3. Other models for interdependent network with intersimilarity

Min et al. [16] investigated the influence of overlapping links on the viability of interdependent networks. Assume that there are some “resource nodes” in a system, and a node is viable only if it can reach a resource node in every layer of the network. They presented two algorithms, called the cascade of activations and deactivations, to obtain the set of viable nodes after initial attack. The results show that overlapping links can facilitate mutual percolation on interdependent networks via the component formed by overlapping links.

Baxter et al. [18] proposed a theory for localized treelike interdependent networks with intersimilarity. They analyzed the situations where a node belongs to the giant mutually connected component, and calculated the size of the mutual component after percolation. They found that correlations caused by overlapped edges can lead to new phase diagrams with multiple and recursive hybrid phase transitions.

### 3.4. Percolation in networks with communities

Networks in nature do not only depend on each other but also have internal community structures [47–49]. Recent research has studied the robustness of networks with communities [19,20,34,50–53]. In Refs. [19,20], the relationship between network community structure and network robustness was studied, where the robustness was characterized by the size of the giant component, and the community structure by the ratio of interconnected nodes between communities or by the strength of the communities.

#### 3.4.1. Percolation in a single network with community

Dong et al. [19] studied the resilience of a single network with communities and found that as the fraction of interconnected nodes (connecting between communities) increases, the whole system becomes more stable. Considering a network composed of two communities, they select a fraction of r nodes in each community as interconnected nodes, which connect to nodes in the other community. When $r > 0$, the size of the giant component at the percolation threshold is greater than 0, that is, the stability of the network increases. Moreover, they find that the effect of the interconnections can be analogized to an external field in magnetic phase transitions, see Fig. 9. These results showed how realistic features in complex infrastructure networks are related to fundamental results from statistical physics. In addition, these results can not only provide guidance for designing more stable systems, but also predict the nature of system failures. Later work expanded this to the case of spatially embedded communities [54] and also interdependent networks [55].

#### 3.4.2. Percolation in interdependent networks with communities

In order to study the relationship between the robustness of interdependent network and network community structure, Sun et al. [20] expressed community structure through the community strength, which is defined by the ratio of internal connections in a community to all

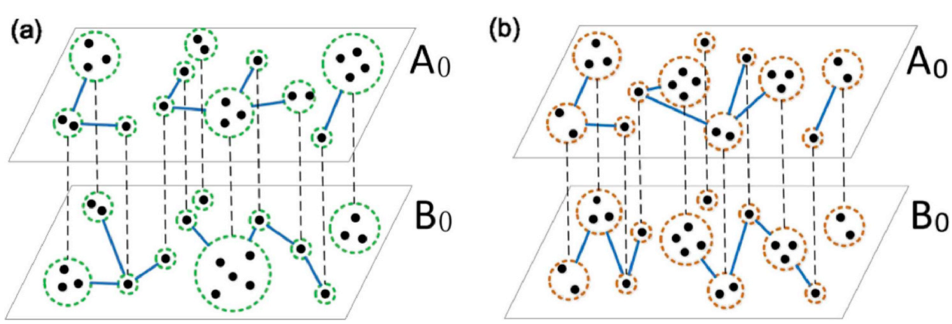


**Fig. 8.** (Color online) **(a)** General group percolation model. In networks $A_0$ and $B_0$, nodes are randomly divided into groups. A group of nodes can be regarded as a supernode. Supernodes in $A_0$ and $B_0$ are full interdependent. **(b)** A special case of the group percolation model is where interdependent supernodes have the same number of regular nodes. After Ref. [15].

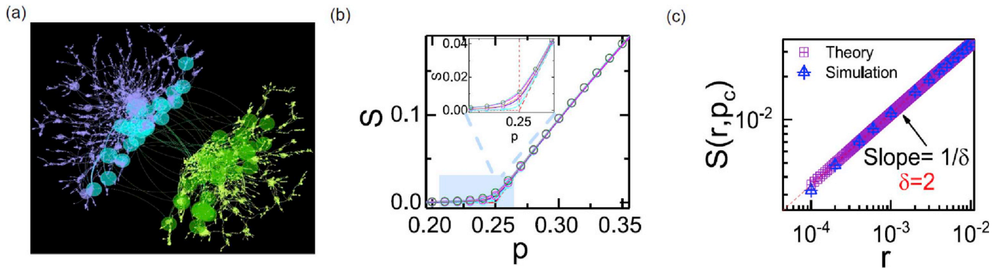


**Fig. 9.** (Color online) **(a)** Coauthor collaboration network with two interconnected communities. **(b)** The relationship between the giant component S and the percolation probability p, for different ratios of interconnected nodes r for ER communities. The red line denotes r = 0, with blue for r = 0.0001 and purple for r = 0.0055. Theory and simulation results are represented by lines and circles, respectively. **(c)** The relationship between the giant component $S(r, p_c)$ at the percolation threshold $p_c$ and the interconnected nodes ratio r. Theory and simulations show that there is a scaling relation $S(r, p_c) \sim r^{1/\delta}$ between them. After Ref. [19].

connections in the same community. Assume that an interdependent network consists of two networks A, B each with two communities, i.e., network A has communities $A_1$, $A_2$, and network B is composed of communities $B_1$, $B_2$. Let $\alpha_{11}$ denote the community strength of $A_1$, with similar definitions of $\alpha_{22}$, $\beta_{11}$, $\beta_{22}$. Setting $\alpha_{11} = \alpha_{22}$ and $\beta_{11} = \beta_{22}$, means that $\alpha_{11}$, $\beta_{11}$ is enough to describe the community structure. Here, we use the percolation threshold $p_{1c}$ of community $A_1$ to indicate the robustness of the entire system. The smaller $p_{1c}$ is, the stronger the system's robustness. Contrary to the single network, stronger community structure does not always increase the robustness of interdependent networks. When $\beta_{11} < 0.436$, $p_{1c}$ becomes larger with the increase of $\alpha_{11}$ as $\alpha_{11} \to 1$. This means high community strength will reduce the system's robustness. When $\beta_{11} = 1$, $p_{1c}$ will increase as $\alpha_{11}$ decreases, see Fig. 10d. This result means that when two communities of network B are not interconnected, network A will face a risk of abrupt collapse.

## 4. Spatial embedding and localized attacks

Many modern infrastructure networks are embedded in two-dimensional space, and the dependency links between the networks are not random but have local constraints [56–58]. Therefore, the cascading failures in spatially-embedded interdependent networks are very different from non-embedded networks [1]. For instance, both the percolation threshold and the type of the percolation transition mainly depend on the length of the dependency links, both under random attack [22,59] and localized attack [23].

### 4.1. Percolation in interdependent embedded networks with limited connectivity link lengths

For an interdependent spatial network, its robustness is determined by the strength of the spatial embedding, which can be characterized by the length of dependency links r [22] or the characteristic connectivity link length $\zeta$ [60]. The smaller is r or $\zeta$, the stronger is the embedding. Simulation results suggest, for intermediate values of the strength of spatial embedding, the percolation transition of the system is first-order and the initial failures will propagate from one region to all regions. Danziger et al. [60] obtained the relationship between the percolation threshold $p_c$ and $\zeta$, finding that the transition shifts from continuous to abrupt with the increase of $\zeta$, see Fig. 11. When $\zeta < \zeta_c$, the percolation

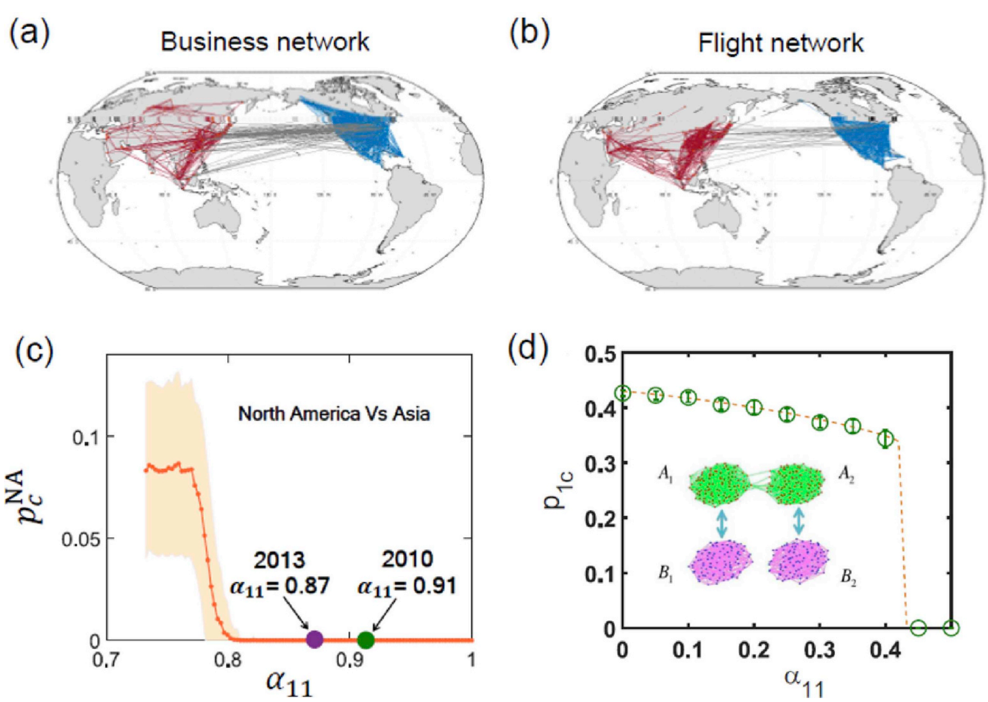


**Fig. 10.** (Color online) **(a,b)** The global business network and business flight network. **(c)** The business network and business flight network in North America and Asia. $\alpha_{11}$ and $p_c^{NA}$ represent the North American business network's community strength and the percolation threshold. The real community strength in 2010 and 2013 is marked by the green and purple points. This suggests the system is evolving towards the critical point where robustness suddenly decreases. **(d)** When network B is fully localized, there is a discontinuous change of $p_{1c}$ with $\alpha_{11}$ between 0 to 0.34. In the figures, the A, B networks each have 10000 nodes, each community has 5000 nodes. 10 simulations are averaged for each data point. After Ref. [20].

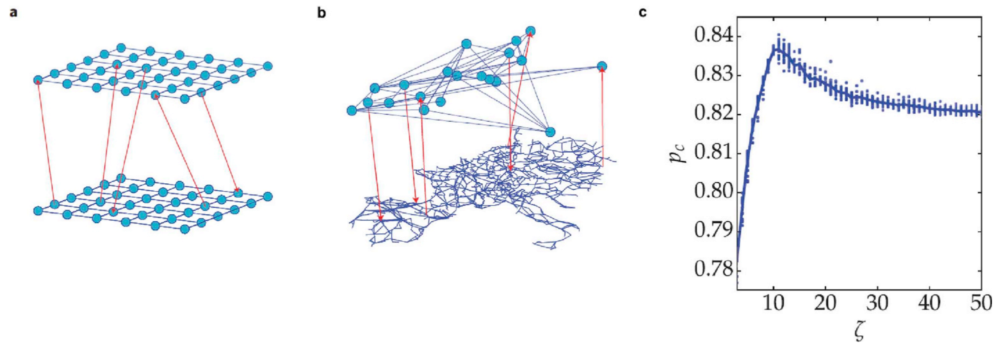


**Fig. 11.** (Color online) **a** The interdependent spatially embedded network model. **b** a real world spatial embedded network. **c** The relationship between the percolation threshold $p_c$ and the strength of the spatial embedding. When $\zeta = \zeta_c$, $p_c$ is maximal. **a,b** After Ref. [61], **c** After Ref. [60].

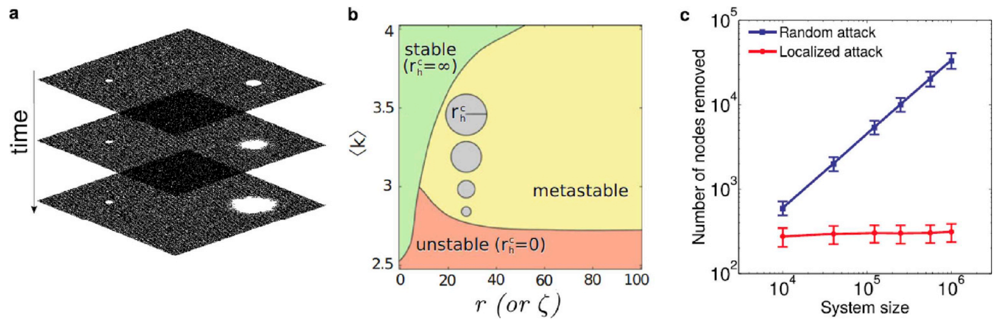


**Fig. 12.** (Color online) **a** The spread of localized failure on two interdependent diluted spatial embedded networks (only one is shown in the graph). The left hole is smaller than $r_h^c$ and the right hole is larger than $r_h^c$. The failure can propagate only when its radius larger than $r_h^c$. **b** The phase graph of interdependent lattice network, the system state is determined by the average degree $\langle k \rangle$ and the maximal dependency link r. In the metastable region, there is a $r_h^c$ which is the minimum radius required for localized failure propagation. $r_h^c$ is only related to $\langle k \rangle$ and r. Note that Vaknin et al. [62] found a very similar phase diagram for multiplex with link length ζ. In this case r is replaced by ζ. **c** The relation between the minimum number of nodes to cripple the system and the system size. After Ref. [63].

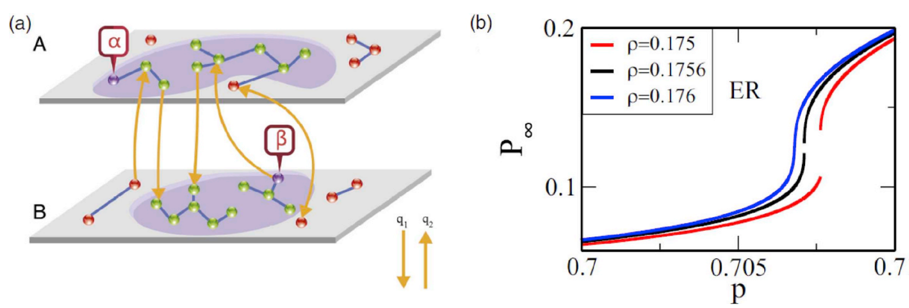


**Fig. 13.** (Color online) **(a)** The model of a two layer interdependent network. Purple points α, β are reinforced nodes. Red points are failed nodes. Yellow arrows represent dependencies between different network nodes. **(b)** In symmetric ER networks with full interdependence, the relation between the functioning component $P_\infty$ and the percolation probability p is shown. Here the average degree is $\langle k \rangle = 4$. ρ is the fraction of reinforced nodes. When ρ approaches $\rho^* = 0.1756$, the system shifts from undergoing an abrupt phase transition to a continuous phase transition. After Ref. [27].

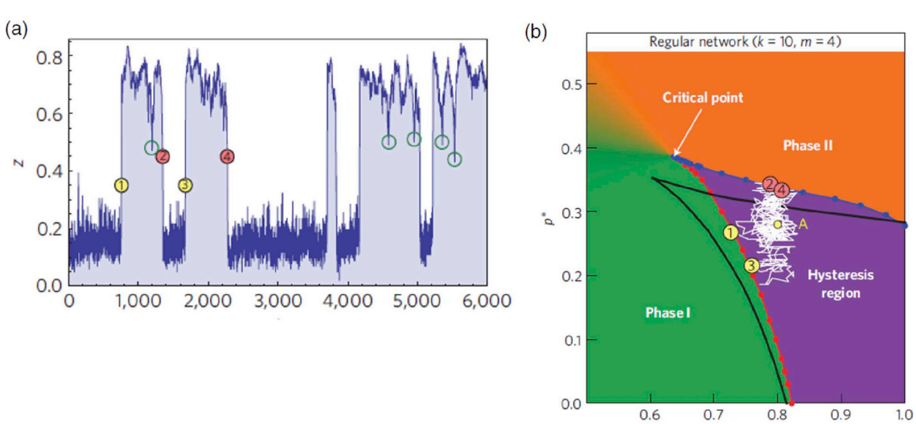


**Fig. 14.** (Color online) **(a)** Changes of system activity z over time. The system demonstrates a phase-flipping phenomenon alternating from functional state I to failed state II. **(b)** The trajectory of the process is shown. By observing points 1–4, we can identify the moment phase-flipping happens where the trajectory crosses the critical line. After Ref. [25].

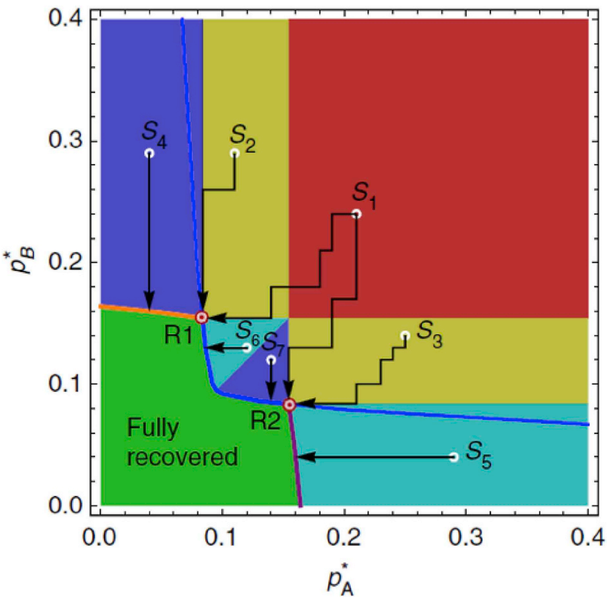


**Fig. 15.** (Color online) Optimal repair for systems in different failed states. The optimal repair path in the $S_1$, $S_2$, $S_3$ regions is closely related to the triple points $R1$, $R2$. In the $S_4$, $S_7$ region, only network B must be repaired. In the $S_5$, $S_6$ region, only network A needs to be repaired. After Ref. [26].

transition is second-order and the percolation threshold $p_c$ increases with $\zeta$. For , the system percolation transition is abrupt and $p_c$ decreases with $\zeta$ until $p_c = 2.4554/\langle k\rangle$ for $\zeta \to \propto$. These results suggest that when the local restriction is strong (high embedding), the system undergoes a continuous percolation transition, which is similar to that of a single-layer network. With the decrease of the strength of spatial embedding, the local restriction is reduced, and the dependency links or connectivity links between the nodes tend towards random. Because of cascading failures, the system undergoes an abrupt percolation transition. In short, the system is in the most vulnerable state when the strength of spatial embedding is at an intermediate value, see the maximal value of $p_c$ in Fig. 11c.

### 4.2. Localized attack on spatially interdependent networks with limited length of dependency links

Failures in spatially embedded systems are often not random but localized. For example, tsunami and earthquakes often cause local damage in real systems, such as the electric power network. Berezin et al. [23] studied the effect of localized attacks on spatially embedded networks with dependencies, and found that localized attack could cause substantially more damage than an equivalent random attack. Furthermore, there is a metastable state: robust to random failure but not to localized failure. More surprisingly, there exists a critical damage size radius $r_h^c$ that is independent of the system size. If the radius of a localized attack, $r_h$, is larger than $r_h^c$, the localized damage will spread and destroy the entire system. Conversely, if $r_h < r_h^c$, the localized attack will not spread. To demonstrate the potential risk of localized attacks in spatially embedded networks, Berezin et al. [23] modeled spatially embedded systems via diluted square lattices of degree $2.5 \le \langle k\rangle \le 4$, and limited the length of dependency links to be less than a distance r. They found that $r_h^c$ is determined by the average degree $\langle k\rangle$ and the maximum length of dependency link r. For a large range of values of r and $\langle k\rangle$, the system is metastable, see Fig. 12. When the system is metastable, the minimum number of nodes whose failure can crash the system under random attacks is proportional to the size of system N. However, this minimum number of nodes remains constant for localized attacks, see Fig. 12c. This means that for a fixed number of failed nodes, the system is robust to random attack for $N \to \infty$, but not to localized attack. It is important to note that Vaknin et al. [62] found a similar phase diagram as Fig. 12b also when the dependency links are of length 0 while the connectivity links are of length $\zeta$.

## 5. Protection strategies

Given that cascading failures can cripple infrastructures, it is necessary to develop protection approaches to avoid a dramatic breakdown and improve stability. Here we review three recently developed theoretical frameworks [25–27], which provide different strategies for preventing the abrupt catastrophic collapse of a single network or an independent network.

### 5.1. Protection by reinforced nodes in interdependent network

In reality, some nodes can be reinforced and can function not only on their own, but also provide support to the entire component connected to them. For instance, when a residential area in a city is facing a sudden blackout, temporary power generation equipment can be used to maintain the use of electricity in this area. In view of this, Yuan et al. [27] introduced the concept of reinforced nodes into interdependent networks. They found that a small fraction of reinforced nodes can prevent abrupt catastrophic collapses. The authors [27] developed a percolation theory that predicts the minimum fraction of reinforced nodes (denoted by $\rho^*$) needed to prevent collapse, see Fig. 13. For n layer symmetric, fully interdependent ER networks, the minimum fraction is $\rho^* = \frac{e^{1-\frac{1}{n}}}{n}$, where $\rho^*$ increases with n. This is consistent with previous results showing that the more layers an interdependent network has, the more vulnerable it is [29]. In particular, for the case of $n = 2$, one needs a fraction of 0.1756 reinforced nodes to avoid catastrophic abrupt collapse.

### 5.2. Spontaneous recovery in a single dynamic network

Another important aspect of infrastructure is the possibility of nodes recovery after some period of time $\tau$. Majdandzic et al. [25] discovered a mechanism of spontaneous recovery after collapse. The failure of a node is assumed to be caused by two reasons: internal failure and external failure. At each time $\tau$, they assume that any node in the network can fail independently of other nodes, with the probability of internal failure given by $p^*$. In addition, a node in a substantially damaged neighborhood will fail externally with probability r. Also, after some characteristic time, nodes can recover. Let the fraction of active nodes z represent the state of the whole network. Fig. 14a demonstrates that introducing dynamic recovery leads to phase-flipping. Phase i represents a high activity state and the low activity state is given by phase *ii*. The dynamic change of the network, given by $p^*$ and r in terms of time t, are marked by $p^*(t)$, $r(t)$. $(p^*(t), r(t))$ and form a trajectory in the phase plane which describes the evolution of the network (see Fig. 14b). When phase-flipping happens, the trajectory will cross the critical line between Phase i or *ii*. For example, the yellow point 1 in Fig. 14a indicates spontaneous recovery, and corresponds to where the trajectory traverses the critical line at the yellow point 1 in Fig. 14b. When the trajectory crosses the critical line and enters Phase i or *ii*, it will rapidly return to the hysteresis region and not cause cascading failure or recovery. Thus, the system will stay in its same phase for some time before switching to the other phase.

### 5.3. Optimal repairing of interdependent networks

In many real infrastructure networks, when the systems fail, they can not only rely on spontaneous recovery, but also need to be repaired manually. However, these networks are usually interdependent. So it's important to find a repair strategy for interdependent networks [26,64]. To improve our understanding of the resilience of infrastructure networks, Majdandzic et al. [26] found an optimal repair strategy in the case of partial or total failures of interconnected networks, that is, they determined the minimum set of nodes, in each network, to be repaired in order to restore the system. Taking two networks A and B for example, the relation between the state of the whole system and the probabilities of internal failures $p_A^*$, $p_B^*$ can be obtained using mean field theory. Each network can be active (state 1) or failed (state 2). Thus, in the case of two networks A and B, the whole system has 4 states (11, 12, 21, 22) and the phase diagram is more complicated than that of a single network. In interdependent networks, except of internal and external failures, there are also interdependent failures, i.e., nodes of a given layer that depend on a failed node from another layer will also fail. If we suppose that only internal failure can be repaired, the optimal repairing strategy is to find the shortest Manhattan distance from the point where the collapsed system is currently situated to the nearest border of the functional 11 region (namely the fully recovered region) as seen in Fig. 15. Majdandzic et al. [26] pointed out that the triple points in the phase diagram play a dominant role in constructing the optimal repairing strategy. The optimal destination point for an initial point in the $S_1$, $S_2$, $S_3$ regions are these triple points ($R_1$ and $R_2$ in Fig. 15). The order of repair (the specific path taken between the initial point and destination point) does not affect the final result. For example, when the system is in the $S_1$ region, in order to find the optimal repair strategy, we need to calculate the Manhattan distance between the initial points and the nearest triple points. In Fig. 15, we see $S_1 \to R_1$, $S_1 \to R_2$. Then we just need to calculate the projection $\triangle p_A^*$, $\triangle p_B^*$ of the Manhattan distance on $p_A^*$ and $p_B^*$ axes to determine the fraction of nodes that need to be repaired in each of networks A and B for which the system of systems will return to functionality.

## Acknowledgments

This work was partly supported by the Israel Ministry of Science and Technology with the Italian Ministry of Foreign Affairs, the Israel Ministry of Foreign Affairs with the Japan Science Foundation, the Israel Science Foundation, the Office of Naval Research Global, Defense Threat Reduction Agency Grant HDTRA-1-10-1- 0014, the Army Research Office, the Binational Science Foundation with the National Science Foundation, and the Bar-Ilan University Center for Cybersecurity and Applied Cryptography project of Guangdong department, No. 2015KQNCX088, No. 2017KZDXM054. YH is supported by the NSFC, Grants No. U1711265 and No. 61773412.

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