No indications of metal-insulator transition for systems of interacting electrons in two dimensions

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The influence of Coulomb interaction on transport properties of spinless electrons in small disordered two-dimensional systems is studied within a tight-binding model. Spatial correlations, inverse participation ratio, and multifractal spectrum of the zero-temperature local tunneling amplitude as well as the dc Kubo conductance are traced as function of the interaction strength U. When U is increased, all of the above quantities are shifted rather smoothly towards localized behavior, indicating the absence of an interaction-driven insulator-metal transition.

Recently, much interest has been focused on the influence of electron-electron interaction (EEI) on the localization properties of two-dimensional electrons in disordered systems. It has been motivated by the experimental observations of a crossover in the behavior of the conductance of low-density two-dimensional electrons from an insulatinglike temperature dependence at low densities to a metallic one at higher densities.[1] In some cases, a transition back to an insulating dependence at even higher densities was observed.[2] This two-dimensional ''metal-insulator'' transition (2DMIT) is in conflict with the scaling theory of localization,[3] which, for noninteracting electrons, asserts that all states in 2D are localized by any amount of disorder. It prompted an intensive theoretical effort including analytical[4] and numerical[5–10] work, which tried to explain the 2DMIT as a result of EEI not taken into account by the scaling theory. Another viewpoint, supported by some recent experiments,[11] maintains that there is no 2DMIT (that is, the systems are insulating at zero temperature) and that the origin of the observed temperature dependence is not a result of a metallic zero-temperature phase.[12]

In the absence of EEI, there are well-established relations between the conductance and the properties of individual single-electron states. Such relations do not exist for the many-electron states. For example, the sensitivity of a single-electron state at the Fermi energy to a change in the boundary condition is a measure of the conductance of the system (the Thouless criteria[13]). Formulation of a similar conjecture for many-electron states is much more intricate.[14,15] Another example is the behavior of the level statistics at the Anderson localization transition. This transition is accompanied by a transition in the single-electron energy-level statistics from Wigner to Poisson,[16] while the many-particle energy-level statistics for noninteracting electrons will remain Poissonian both in the metallic and in the insulator regime.[17]

In previous studies of 2D spinless electrons with EEI,[5–9] numerous equilibrium properties such as many-particle energy-level statistics, two-electron localization length high above the Fermi energy, persistent current flow patterns, and charge-density response to an external perturbation were studied. As a function of the interaction strength U, the footprints of some of these quantities undergo qualitative changes. It might then be tempting to interpret it as a evidence of a 2DMIT. However, the properties listed above are not directly related to zero-temperature transport properties of the electron system, which are naturally measured in experiments. It is therefore mandatory to carefully examine some quantities that are directly related to transport properties of the system. Since the problem under study involves disorder and EEI, the region of interest in parameter space is that for which both of them are significant. This excludes the possibility of using perturbation theory, and leaves us with the necessity of employing exact numerical diagonalization.

This task, which is feasible for relatively small systems, is carried out in the present paper. The statistical properties of tunneling amplitudes and the conductance distribution of the system are examined. It is shown that EEI systematically *attenuates* the transport through the system and enhances its insulating features. Thus, although EEI may significantly alter some of the properties of a 2D electron system as demonstrated in previous studies,[6–9] it is conjectured here that there is no numerical evidence that it can drive a transition in the transport properties of spinless electrons.

We consider systems composed of four, six, and eight interacting electrons residing on 6×6, 5×5, and 5×4 lattices (correspondingly) having a torus geometry. It is of course not possible here to directly mimic the experimental procedure in which the conjectured 2DMIT is driven through variation of electron density. Instead, the physical content of this density variation can be captured by controlling the ratio of the Fermi energy to the interaction energy. In the present model, it is achieved simply by changing the interaction strength U while keeping other parameters intact.[7]

In the tight-binding approximation, the Hamiltonian of the system is given by

$$H = \sum_{k,j} \epsilon_{k,j} a_{k,j}^{\dagger} a_{k,j} - V \sum_{k,j} (a_{k,j+1}^{\dagger} a_{k,j} + \text{H.c.})$$

$$- V \sum_{k,j} (a_{k+1,j}^{\dagger} a_{k,j} + \text{H.c.}) + U \sum_{k,j > l,p} \frac{a_{k,j}^{\dagger} a_{k,j} a_{l,p}^{\dagger} a_{l,p}}{|\vec{r}_{k,j} - \vec{r}_{l,p}|/s},$$

$$(1)$$

where $\vec{r}=(k,j)$ denotes a lattice site, $a^{\dagger}_{k,j}$ is an electron creation operator, $\epsilon_{k,j}$ is the site energy, chosen randomly between $-W/2$ and $W/2$ with uniform probability, V is a constant hopping matrix element, and s is the lattice constant. Using the Lanczos method, we obtain the many-particle eigenvalues ε^{N}_{α} and eigenfunctions $|\alpha^{N}\rangle$, where N is the number of electrons. The zero-temperature local tunneling amplitude $\langle 0^{N}|a^{\dagger}_{r}|0^{N-1}\rangle$ between the ground state of N and $N-1$ electrons can be employed here in order to characterize the transport properties of the many-particle interacting system. It has the advantage that only the ground-state energy and eigenvector for N and $N-1$ electrons need to be calculated. The use of the tunneling amplitude in this context can be motivated and substantiated by the following considerations: In the independent-particle approximation, the tunneling density of state (TDOS) is given by

$$\nu(\varepsilon)=\sum_{n}\int|\psi_{n}(\vec{r})|^{2}dr\,\delta(\varepsilon-\varepsilon_{n})=\sum_{n}\,\delta(\varepsilon-\varepsilon_{n}),\quad(2)$$

where ψ_{n} is the nth single-particle eigenvector and ε_{n} is the nth single-electron eigenvalue. For the many-body interacting system, the TDOS is defined as[18]

$$\nu(\varepsilon)=\sum_{\alpha}\left|\left\langle\alpha^{N}\left|\sum_{\vec{r}}a^{\dagger}_{r}\right|0^{N-1}\right\rangle\right|^{2}\delta(\varepsilon-(\varepsilon^{N}_{\alpha}-\varepsilon^{N-1}_{0})).$$
$$(3)$$

The conductance $\sigma(\varepsilon)$ is related to the transmission $t(\vec{r},\vec{r}',\varepsilon)$ of an electron with energy ε between two points $\vec{r}$, $\vec{r}'$ on the interface of the system with external leads through the Landauer formula[19] $\sigma(\varepsilon)=(e^{2}/h)\Sigma_{\vec{r},\vec{r}'}|t(\vec{r},\vec{r}',\varepsilon)|^{2}$, where the sum is over all points on the interface. For a noninteracting system, the transmission is expressible in terms of single-particle wave functions,

$$t(\vec{r},\vec{r}',\varepsilon)=\sum_{n}\psi^{*}_{n}(\vec{r})\psi_{n}(\vec{r}')\delta(\varepsilon-\varepsilon_{n}).\qquad(4)$$

As discussed in Ref. 20, an appropriate Landauer formula connecting transmission and conductance for an interacting system coupled to external leads has a similar structure, in which the transmission in the interacting region is given by

$$t(\vec{r},\vec{r}',\varepsilon)=\sum_{\alpha}\langle\alpha^{N}|a^{\dagger}_{r}|0^{N-1}\rangle\langle 0^{N-1}|a_{\vec{r}'}|\alpha^{N}\rangle$$
$$\times\delta(\varepsilon-(\varepsilon^{N}_{\alpha}-\varepsilon^{N-1}_{0})).\qquad(5)$$

In the absence of interaction, Eq. (3) is reduced to Eq. (2), and Eq. (5) becomes identical to Eq. (4). Indeed, the tunneling amplitude $\langle\alpha^{N}|a^{\dagger}_{r}|0^{N-1}\rangle$ couples only many-particle states different from each other by the addition of a single particle to an unoccupied single-particle state. Thus, for a many-particle state $|\alpha^{N}\rangle$ that corresponds to a Slater determinant in which all $N-1$ single-particle state are occupied as well as the nth single-particle state above the Fermi energy, one obtains $\psi_{n}(\vec{r})=\langle\alpha^{N}|a^{\dagger}_{r}|0^{N-1}\rangle$ and $\varepsilon^{N}_{\alpha}-\varepsilon^{N-1}_{0}$

$=\varepsilon_{n}$, while for other many-particle states the matrix element vanishes. This behavior suggests that the tunneling amplitude is the appropriate quantity to replace the single-electron wave function in studying transport properties of interacting systems. A similar procedure is employed in Ref. 21 in order to generalize the concept of inverse participation ratio for interacting systems.

Note, however, that once interactions are present, a many-particle state is a superposition of many different Slater determinants, and, generically, $\Sigma_{\vec{r}}|\langle 0^{N}|a^{\dagger}_{r}|0^{N-1}\rangle|^{2}\neq 1$, where for states corresponding to quasiparticles the matrix elements are dominant.[22] Thus, for interacting systems the tunneling amplitude $\langle 0^{N}|a^{\dagger}_{r}|0^{N-1}\rangle$ is not normalized. The reason is that, due to interaction, the basic objects are quasiparticles (rather than particles) with a finite lifetime. In this context, interaction affects transport properties of a given system in two different ways. The first (which is not related at all to quantum localization) is implied by variation of the density of states at a given energy, while the second one (which is the essence of quantum localization) is manifested through a change in the correlation properties of the tunneling amplitude between different points. Therefore, in order to study the influence of EEI on quantum localization, it is useful to define an effective tunneling amplitude $\phi(\vec{r})$ $=\langle 0^{N}|a^{\dagger}_{r}|0^{N-1}\rangle/(\Sigma_{\vec{r}}\langle 0^{N}|a^{\dagger}_{r}|0^{N-1}\rangle^{2})^{1/2}$, which will be used to study transport properties of an interacting system.

The effective tunneling amplitude $\phi(\vec{r})$ of the system introduced above is calculated for several values of disorder, and then analyzed by employing well known procedures for elucidating properties of single-particle wave functions such as spatial correlation function, inverse participation ratio, and a multifractal analysis. Results for the correlation function

$$C(r)=\langle|\phi(\vec{r})\phi(\vec{r}')|\rangle_{|\vec{r}-\vec{r}'|=r}\qquad(6)$$

for the 6×6, four-electron system averaged over 100 realizations are shown in Fig. 1 (similar results are obtained for
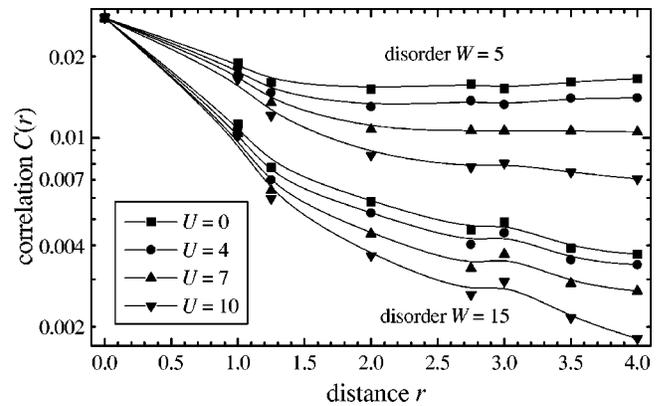

FIG. 1. The decay of the correlation function $C(r)$ for $\phi(\vec{r})$ for two different disorder strengths $W=5$ (in units of V) in the upper part and $W=15$ in the lower part and four interaction strengths (see legend). The correlation function decays faster for stronger disorder and for stronger interaction.
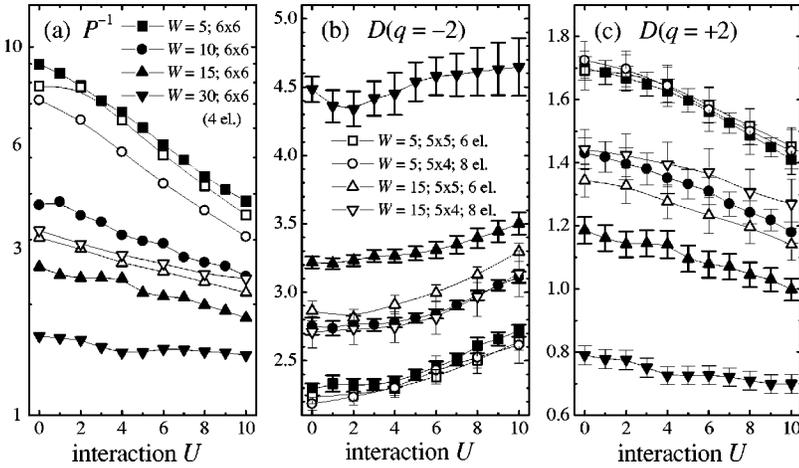
FIG. 2. (a) Participation ratio P^{-1} as a function of interaction strength U for 6×6, four electrons; 5×5, six electrons; 5×4, eight electrons; and different disorder values (all averaged over 100 realizations of disorder). Parts (b) and (c) show the results $D(-2)$ and $D(+2)$ from a multifractal analysis. The results indicate a monotonous crossover to stronger localized states with increasing interaction or disorder.

the other system sizes). Evidently, $C(r)$ decays as a function of r indicating the loss of amplitude correlations. The decay is faster for stronger disorder as is the case for a system of noninteracting electrons. Moreover, the decay of $C(r)$ evolves smoothly with increasing interaction strength U, indicating stronger localization at higher values of U, showing no sign of a 2DMIT or an intermediate metallic phase. These results are significant, since an identical system to that considered here undergoes a transition in the character of its persistent current at intermediate values of U.[23,7] In particular, the flow pattern of the local persistent current is shown to be ordered and essentially one-dimensional. Our new result indicates that this kind of behavior should not be interpreted as a signature of a delocalized (Coulomb) phase.

A similar feature is exposed in the inverse participation ratio P^{-1} with $P = \sum_{\vec{r}} |\phi(\vec{r})|^4$. Figure 2(a) shows the monotonous decrease of P^{-1} with increasing U and disorder W. Again, there is no sign of an intermediate metallic phase.

In order to further elucidate the characteristics of the effective tunneling amplitude $\phi(\vec{r})$, we consider its multifractal pattern (for a review, see Ref. 24). The multifractal exponents $\tau(q)$ are obtained from the scaling behavior of the partition function

$$Z_q(l) = \sum_i [p_i(l)]^q \sim l^{\tau(q)}, \tag{7}$$

where the sum runs over all boxes i of linear size l and $p_i(l)$ are the probability densities in each box. Figures 2(b) and 2(c) show $D(q) = \tau(q)/(q-1)$ for $q = -2$ and $+2$ as a function of interaction strength U. Again, we do not find any indication of a 2DMIT.

Another way to present the results of the multifractal analysis is the singularity spectrum $f(\alpha)$, which is the Legendre transform of $\tau(q)$. For each singularity exponent α, $f(\alpha)$ can be interpreted as the fractal dimension of the subset of boxes i, which are characterized by $p_i(l) \sim l^\alpha$. Figure 3 shows the $f(\alpha)$ spectra for the 6×6, four-electron system at two disorder strengths. Since the singularity spectra are not independent of interaction strength (or system size), the multifractality is not an indication of a critical point. The spectra become wider with increasing interaction

or disorder, indicating stronger localized states rather than generic multifractal states. Yet the bottom line is that we do not find any indication of a metallic phase but a mere monotonous crossover into the regime of stronger localization.

In order to relate our results for the tunneling amplitude to the standard Kubo formulation of the conductance, we determine the distribution of the conductances σ for different disorder configurations and, further, study the dependence of the average conductance on the interaction strength U. The conductance at zero temperature is calculated using the many-particle Kubo formula[14]

$$\sigma = \left(\frac{\pi e^2}{Mh}\right) \sum_\alpha' \frac{|\langle \alpha^N | J_x | 0^N \rangle|^2 \varepsilon_{\alpha,0}^N \gamma}{\varepsilon_{\alpha,0}^{N\,2} + \gamma^2}, \tag{8}$$

where $\varepsilon_{\alpha,0}^N = \varepsilon_\alpha^N - \varepsilon_0^N$, J_x is the current operator, and M is the number of sites. The inelastic broadening γ is chosen to be of the same order as the rescaled ''single-electron'' level separation defined as $B/N(M-N)$, where B is the width of the many-particle energy band.[25,9] The calculation of the conductance is cumbersome since it requires the computation of many-body eigenfunctions for all the low-lying excita-
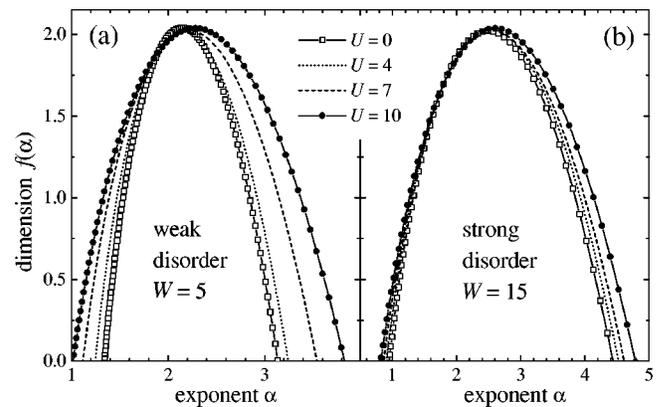

FIG. 3. Singularity spectra for effective multiparticle transmission functions $\phi_{\vec{r}}$. For each singularity exponent α, $f(\alpha)$ can be interpreted as the fractal dimension of the subset of boxes i, which are characterized by $p_i(l) \sim l^\alpha$. The spectra become wider with increasing interaction or disorder indicating a crossover to stronger localized states.
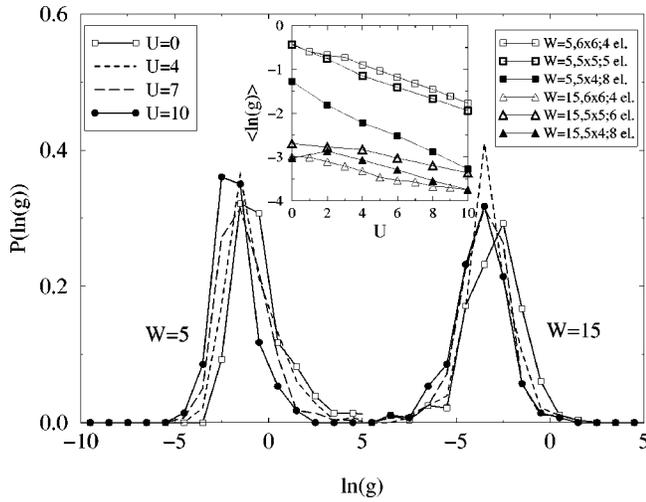
FIG. 4. The log of the dimensionless conductance distribution for different values of disorder and interaction strength for the 6 ×6, four electron system. Inset: the average of the log of the dimensionless conductance as a function of the interaction for the 6 ×6, four electron system (averaged over 280 realizations of disorder); 5×5, six electrons (60 realizations); and 5×4, eight electrons (100 realizations).

tions (20 in our calculation), which is a difficult numerical task even within the Lanczos algorithm.

We first present the behavior of the conductance as a function of U. For the 6×6, four-electron system, the distributions of the dimensionless conductance $g=(h/e^2)\sigma$ are shown in Fig. 4 for two different values of disorder ($W=5, W=15$). For the weaker disorder, which for this small system corresponds in the noninteracting case to a marginal

metal,[7] the distribution approaches a log-normal distribution as interaction strength increases. For the stronger disorder, which even for the noninteracting case is in the localized regime, the log-normal distribution is preserved for any strength of interaction. Since the conductance distribution of a localized system is expected to follow a log-normal distribution,[26] while a metallic system should follow a normal distribution, this behavior indicates that repulsive electron-electron interactions suppress the metallic characteristics of the Kubo conductance. This is consistent with results reported some time ago pertaining to the influence of EEI on the conductance.[14,27] In particular, it has been shown that in the diffusive regime the average conductance is suppressed by interactions, while only deep in the localized regime, where the average current is very small, is some enhancement possible.[27]

The same trend can be seen when inspecting the average logarithmic conductance, displayed in the inset of Fig. 4. Thus, neither the average conductance nor its distribution show any traces of a 2DMIT.

In conclusion, we have investigated the transport properties of spinless electrons in disordered clusters. Neither the tunneling amplitude nor the conductance show any evidence of a 2DMIT. It has been shown that the tunneling amplitude is the natural analog of the single-electron wave function appropriate for interacting many-particle systems. This is quite attractive, since numerous methods developed to connect the properties of single-electron wave functions to transport properties of the system may now be efficiently applied to the tunneling amplitude function.

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