Nonuniversal transport exponents in quasi-one-dimensional systems with a power-law distribution of conductances

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We study transport in quasi-one-dimensional systems consisting of n connected parallel chains of length L with a power-law distribution of bond conductivities $P(\sigma)\sim\sigma^{-\alpha}$, $\alpha<1$, $\sigma\leq 1$. When the transverse bonds are perfect conductors, we find that the conventional law for the transport exponents in one-dimensional systems is not universal but depends sensitively on n. For n finite, there exists a critical value of α, $\alpha_c=1-1/n$. For $\alpha\leq\alpha_c$, the resistivity exponent $\bar{\zeta}$ and the diffusion exponent d_w stick at their classical values $\bar{\zeta}=1$ and $d_w=2$. For $\alpha>\alpha_c$, both exponents vary continuously with n: $\bar{\zeta}=1/n(1-\alpha)$ and $d_w=1+1/n(1-\alpha)$. These values represent lower bounds if the transverse bonds have the same power-law distribution. In the case of $n=1$, the transport exponents accept their well-known one-dimensional values. In the two-dimensional limit $n\sim L$, we obtain $\bar{\zeta}=0$ and $d_w=2$, irrespective of α.

In recent years, the problem of transport in one-dimensional systems with a broad distribution of resistances has been studied extensively (see, e.g., Refs. 1–4). For a power-law distribution of bond conductivities

$$P(\sigma)\sim\sigma^{-\alpha},\ \alpha<1,\ \sigma\leq 1\ , \tag{1}$$

the transport exponents d_w and $\bar{\zeta}$ depend sensitively on α. The exponents d_w and $\bar{\zeta}$ are defined by $\langle x^2\rangle\sim t^{2/d_w}$ and $\rho\sim L^{\bar{\zeta}}$, where $\langle x^2\rangle$ is the mean-square displacement of a random walker, ρ the resistivity, and L the system length. It was found that $\bar{\zeta}$ and d_w vary with α as[1–4]

$$\bar{\zeta}=\begin{cases}1 \text{ for } \alpha\leqq 0\ ,\\ \dfrac{1}{1-\alpha} \text{ for } \alpha>0\end{cases} \tag{2a}$$

and

$$d_w=\begin{cases}2 \text{ for } \alpha\leqq 0\ ,\\ \dfrac{2-\alpha}{1-\alpha} \text{ for } \alpha>0\ .\end{cases} \tag{2b}$$

The theoretical results (2a) and (2b) were found useful to describe several physical systems. For example, (I) the temperature dependence of the dynamical conductivity exponent observed in the one-dimensional superionic conductor hollandite can be understood[4] from Eq. (2). (II) Continuum random systems such as the random-void model can be mapped[5] onto random percolation networks with the distribution (1) of bond conductivities. Employing the "one-dimensional" nature of the backbone of the percolation cluster, bounds for the transport exponents have been derived[3,5–8] from (2). (III) The problem of biased diffusion in random structures such as those described in Refs. 9 and 10 can be mapped on biased diffusion in a linear chain with a power-law distribution of transition rates. In this case, the parameter α depends on the bias field and a dynamical phase transition occurs. (IV) Anomalous relaxation in spin glasses can be interpreted in terms of stochastic motion (in phase space) with a power-law distribution of transition rates.[11,12]

In this paper we study how the transport properties are affected by the power-law distribution (1) if the system of interest is quasi-one-dimensional, consisting of n connected parallel linear chains (see Fig. 1) of length L. For finite n, in the limit $L\rightarrow\infty$, the system is one dimensional. However, we will show that both $\bar{\zeta}$ and d_w depend sensitively on n and thus (2) is not universal for one-dimensional systems.

We consider the case where the horizontal bonds have conductances $\sigma_{i,j}$ which are distributed according to Eq. (1). The index j labels the chains, $j=1,2,\ldots,n$, while the index i labels the bonds along one chain, $i=1,2,\ldots,L$. For simplicity we first assume that the vertical bonds are perfect conductors. The total horizontal conductivity Σ_n is

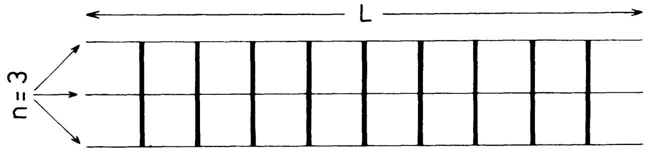

FIG. 1. A quasi-one-dimensional system of length L consisting of $n=3$ connected linear chains. The vertical bonds are perfect conductors while the horizontal bond conductances are chosen from the distribution (1).

given by Kirchhoff's law

$$\Sigma_n^{-1}=\sum_{i=1}^{L}\frac{1}{\sigma_{i,1}+\sigma_{i,2}+\cdots+\sigma_{i,n}} , \quad (3)$$

where $\sigma_{i,1}+\sigma_{i,2}+\cdots+\sigma_{i,n}\equiv\Sigma$ is the conductivity of the n parallel conductors in the ith segment of the chain.

For $n=2$, the singular part of the distribution of conductivities $\Sigma=\sigma_{i,1}+\sigma_{i,2}$ is given by

$$P_2(\Sigma)=\int_0^{\Sigma}\frac{d\sigma}{\sigma^{\alpha}(\Sigma-\sigma)^{\alpha}}=B(1-\alpha,1-\alpha)\Sigma^{1-2\alpha} , \quad (4)$$

where $B(x,y)$ is the β function. The generalization of Eq. (4) for n parallel conductors is

$$P_n(\Sigma)\sim\prod_{k=1}^{n-1}B(k(1-\alpha),1-\alpha)\Sigma^{n-1-n\alpha}\sim\Sigma^{-\tilde{\alpha}} , \quad (5)$$

where $\tilde{\alpha}=n\alpha-(n-1)$. We must distinguish between two regimes of α. For $\alpha\leq\alpha_c=1-1/n$, $\tilde{\alpha}$ is negative and the sum in (3) is linear in L; we recover the conventional result for uniform bond conductivities

$$\Sigma_n^{-1}\sim L/n . \quad (6)$$

For $\alpha>\alpha_c$, $\tilde{\alpha}$ positive, one can use Eq. (2) by substituting $\tilde{\alpha}$ instead of α. Thus, for finite n the corresponding conductivity exponent $\tilde{\zeta}$, $\Sigma_n^{-1}\sim L^{\tilde{\zeta}}$ is given by

$$\tilde{\zeta}=\begin{cases}1 \text{ for } \alpha\leq\alpha_c , \\ 1/n(1-\alpha) \text{ for } \alpha>\alpha_c .\end{cases} \quad (7a)$$

The corresponding diffusion exponent d_w is

$$d_w=\begin{cases}2 \text{ for } \alpha\leq\alpha_c , \\ 1+1/n(1-\alpha) \text{ for } \alpha>\alpha_c .\end{cases} \quad (7b)$$

Consequently, only for the special case $n=1$ do our results for Eq. (7) reduce to the well-known[1] relations (2). For n finite, the system is still one dimensional, but the transport exponents depend on n and therefore they are not universal. The physical reason for the n dependence can be understood as follows. The dominant contributions to the total resistivity in Eq. (3) come from those terms where in one column i *all* conductances are close to their minimum value. If, for example, in one column of n conductors $n-1$ conductors have very small conductivity but one has unit conductivity, then the current will predominantly flow along the good conductor and the resulting conductivity of the considered column will be governed by the good conductor. The probability that all conductors in one column have small conductances decreases if the number n of the conductors in one column increases. Therefore, for the

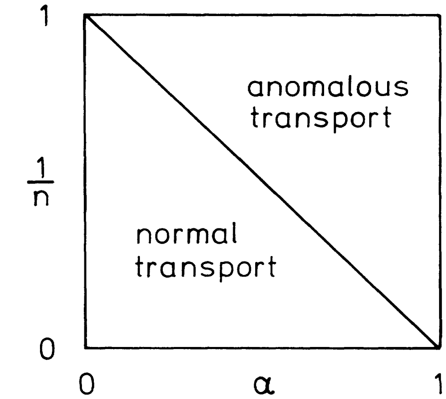

FIG. 2. Phase diagram of the dynamical phase transition in the (α,n) space. Below the critical line transport is normal, while above the critical line transport is anomalous.

same value of α, the transport exponents $\tilde{\zeta}$ and d_w decrease with increasing n. Correspondingly, the critical value α_c above which anomalous transport occurs, increases monotonically with increasing n (see Fig. 2).

In the two-dimensional limit $n\sim L$, there is no critical value of α and we find the normal transport behavior of two-dimensional systems, i.e.,

$$\tilde{\zeta}=0, \; d_w=2 , \quad (8)$$

irrespective of α. This result follows directly from Eq. (6).

So far, for deriving our main results (6) and (7), we have considered a system of vertical and horizontal bonds where only the horizontal bond conductances were chosen from a power-law distribution (1), but the vertical bonds were perfect conductors. By distributing the horizontal conductors according to Eq. (1) the conductivity of the system can only decrease. Thus our results for $\tilde{\zeta}$ and d_w are rigorous lower bounds for the general case in which all bonds horizontal and vertical are distributed according to (1).

Our results can be applied to quasi-one-dimensional random mixtures of singly and multiply connected bonds. If the concentration c of singly connected bonds is finite, then according to (7) the singly connected bonds dominate the transport behavior and the exponents are given by Eq. (2). It is interesting to note that also in more complicated random systems, such as the percolation backbone, the singly connected bonds seem to dominate the transport for large values of α.[8,13,14]

Note added in proof. After this manuscript had been submitted we learned that related results have been derived by I. Webman (unpublished).

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