Phase Transition in Diffusion-Limited Aggregations

In a recent Letter, Lee and Stanley[1] enumerated exactly all configurations of diffusion-limited aggregations (DLA) in square lattices of up to 5×5 sites. They found evidence for a phase transition in the multifractal spectrum of the growth probabilities. The phase transition shows up in the analogs of the "thermodynamical" quantities such as the free energy, entropy, and specific heat. Of particular interest is the sharp peak found in the specific heat around the critical moment ("critical inverse temperature") $\beta_c = -1.0$.

According to Lee and Stanley,[1] the partition function is defined as

$$Z(\beta,L) = \sum_\alpha C_\alpha \sum_i p_{i,\alpha}^\beta , \tag{1}$$

where $p_{i,\alpha}$ is the growth probability of the perimeter site i of configuration α, and C_α is the weight of configuration α. L denotes the system size. From (1) the free energy F is obtained,

$$F(\beta,L) = -\frac{\ln Z(\beta,L)}{\ln L} . \tag{2}$$

The energy E and the specific heat C represent the first and second derivatives of F with respect to β, $E(\beta,L) = \partial F(\beta,L)/\partial\beta$ and $C(\beta,L) = -\partial^2 F(\beta,L)/\partial^2\beta$.

In this Comment we present numerical results which support the intriguing findings of Lee and Stanley. We considered larger systems but averaged only over *one* (typical) DLA cluster. In contrast, Lee and Stanley considered rather small systems but averaged over *all* configurations, including the very rare ones.

We considered a single DLA cluster of 1000 sites and determined numerically the stationary solution of the diffusion equation,[2] from which we obtained the growth probabilities p_i and the related "thermodynamic" quantities. For a single configuration, $\alpha = C_\alpha = 1$.

Our results for the energy and the specific heat are shown in Fig. 1. Despite the different type of averaging and the different system size they resemble very much the results [Figs. 5(a) and 5(b)] presented in Ref. 1 and support strongly Lee and Stanley's prediction of a phase transition in the multifractal spectrum of DLA. The actual position of the critical point is different in our calculations. We find $\beta_c \simeq -0.4$, lower than the value $\beta_c \simeq -1$ found in Ref. 1. We regard the question whether the critical point depends on the type of averaging or whether it tends to zero in the thermodynamic limit or stays at a finite negative value as an interesting open question which will receive an answer in the near future.

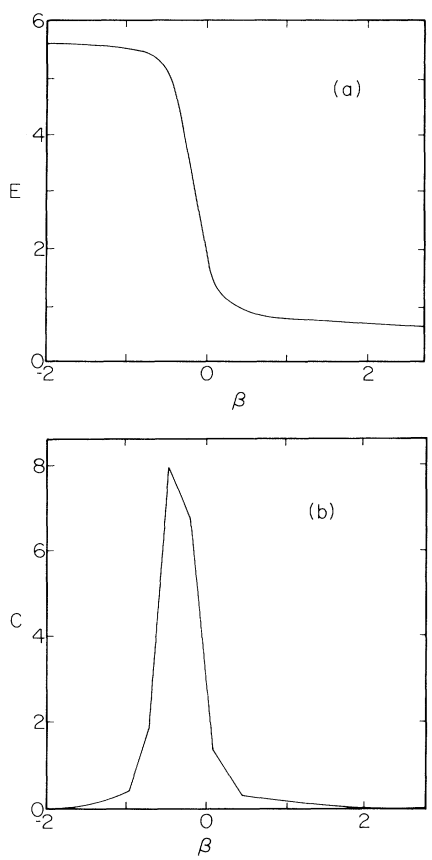

FIG. 1. (a) Energy $E(\beta,L)$ and (b) specific heat $C(\beta,L)$ as a function of β for a single DLA cluster of 1000 sites. Note that, in particular, the shape of the specific heat is surprisingly similar to the shape of $C(\beta,L)$ in Ref. 1.

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