# Analysis of daily temperature fluctuations 

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#### Abstract

We study daily temperature fluctuations over more than 50 yr in two places on the globe that are separated by more than 3000 km . We analyze the temperature fluctuations $\Delta T_{i}$ with respect to the mean noon temperature $\left\langle T_{i}\right\rangle$ averaged, for each day of the year, over the whole year, $\Delta T_{i}=T_{i}-\left\langle T_{i}\right\rangle$. We find that the $\Delta T_{i}$ are correlated and can be characterized for up to at least $10^{3}$ days by a power law correlation with an exponent $\alpha \cong 0.65$.


In recent years it was found that many complex systems in nature display anomalous fluctuations characterized by long range power-law correlations [1-6]. Prominent recent examples include DNA sequences and heartbeat intervals [6-9]. The main difficulty in detecting these correlations is the nonstationarity nature of the data. Several methods have been applied to overcome this difficulty including the Detrended Fluctuation Analysis (DFA) [10] and the wavelet [11] methods.

In this paper we consider temperature fluctuations and address the question, whether the daily temperature variations are correlated and can be characterized by power-law correlations. Of particular interest is the characteristic time in which the temperature variations are correlated.

To deal with these questions, we have analyzed daily noon temperature data (at noon) taken from two weather stations (Pendleton and Huron) in the USA. Both stations are more than 3000 km apart, Pendleton is located in Oregon, while Huron is in Michigan. The data for Pendleton are from 1 January 1938 till 31 December 1994, while the data for Huron are from 1 January 1940 till 31 December 1994. All temperature data are measured in the Fahrenheit scale.

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Fig. 1. The temperature landscape $y(n)$, Eq. (2), for data taken from two weather stations: (a) Pendleton and (b) Huron.

To overcome the natural nonstationarity of the temperature data due to season trends, we have studied the variations of the daily noon temperature with respect to the mean daily noon temperature $\left\langle T_{i}\right\rangle$ averaged over all the 57 and 55 yr , respectively:

$$
\begin{equation*}
\Delta T_{i}=T_{i}-\left\langle T_{i}\right\rangle \tag{1}
\end{equation*}
$$

To overcome linear trends left in the system (some years could be warmer or colder than the average year), we apply a DFA-type method, which is described below.

For analyzing the correlations in $\Delta T_{i}$ we first plot, in Fig. 1, the function

$$
\begin{equation*}
y(n)=\sum_{i=1}^{n} \Delta T_{i} \tag{2}
\end{equation*}
$$

which can be viewed as a landscape of the temperature fluctuations. The parameter $n$ is the number of days which runs, for example in the Pendleton case, from $n=1$ to $n=N=57 \times 365=20805 \mathrm{~d}$.

Next, we divide the abscissa ( $n$-axis) into equal intervals of length $l$, i.e., into $N / l$ intervals. In each interval we calculate the squared fluctuations $F^{2}(l)$ of $y(n)$ with respect to a straight line, $z(n)=a n+b$, connecting the two values of $y_{n}$ at the end points of the interval,

$$
\begin{equation*}
F^{2}(l)=(1 / l) \sum_{n=k l+1}^{(k+1) l}(y(n)-z(n))^{2}, \quad k=0,1,2, \ldots,(N / l-1) \tag{3}
\end{equation*}
$$

Averaging $F(l)$ over the $N / l$ intervals gives the mean temperature fluctuations $\langle F(l)\rangle$ as a function of $l$.

If the $\Delta T_{i}$ were random uncorrelated variables or short range correlated variables, we would expect

$$
\begin{equation*}
\langle F(l)\rangle \sim l^{x} \tag{4}
\end{equation*}
$$

with $\alpha=\frac{1}{2}$. An exponent $\alpha \neq \frac{1}{2}$ in a certain range of $l$ values suggests the existence of a power-law long range correlations in that range.

Fig. 2 shows $\langle F(l)\rangle$ for the two temperature landscapes shown in Fig. 1. It is seen that for both temperature landscapes, the slope representing the exponent $\alpha$ is about $\alpha \cong 0.65$ starting from about $l=20 \mathrm{~d}$. From the data, we cannot see where this power law ends. The power law extends to at least $10^{3} \mathrm{~d}$. This suggests the existence of long-range power-law correlations in the daily temperature fluctuations up to at least three years. It is possible that the correlations extend much further. However, for time scales above $10^{3} \mathrm{~d}$ the data start to scatter, and we cannot rule out the possibility of a smaller exponent including $\alpha=\frac{1}{2}$.

We also tested how sensitive our result is to the assumption of the mean daily noon temperature as a reference frame. For this purpose we replaced $\left\langle T_{i}\right\rangle$ in Eq. (1) (a) by the daily noon temperature of a given (typical) year, and (b) by the mean daily noon temperature of the first and the last five years and repeated all the analysis of Eqs. (1)-(4). We found that the results for $\langle F(l)\rangle$ are very similar to those of Fig. 2 supporting the previous results.

To summarize, we have presented an analysis of daily noon temperature fluctuations from two weather stations. Our results suggest the existence of long-range power law correlations in weather fluctuations in the range of at least 3 yr. However, we consider our results as preliminary, since we considered only two places on the globe. More


Fig. 2. Plot of the mean temperature fluctuations $\langle F(l)\rangle$ versus the interval length $l$ on a double-logarithmic scale for the two daily temperature landscapes shown in Fig. 1. The straight line has a slope of 0.65 .
data and longer periods of times are needed to obtain more conclusive results and to confirm our finding.

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