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Anderson localization in a random correlated energy landscape

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Abstract

We study the localization of electronic wave functions in the one-dimensional Anderson model with diagonal disorder, where the site energies are long-range correlated. We find different behavior at the band edge and at the band center. Close to the band edge, the correlations lead to a decrease in the localization lengths. At the band center, in contrast, the localization length is drastically enhanced by the correlations. Using level statistics we find strong evidence that also these states remain localized.

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Eigenstates of the one-dimensional Schrödinger equation in a random potential are usually localized [1,2]. Exceptions have been found, however, [3–5], indicating that localization in $d=1$ is not a general phenomenon but depends on the specific structure of the random system.

Here, we study wave functions in the Anderson model with diagonal disorder in $d=1$. In the tight-binding approximation, the Schrödinger equation becomes

$$E\psi_n = \lambda V_n \psi_n - \psi_{n+1} - \psi_{n-1}, \tag{1}$$

where E is the eigenvalue, $|\psi_n|^2$ is the probability amplitude of the wave function for site n, and λV_n is the local potential. V_n are random numbers distributed in the interval

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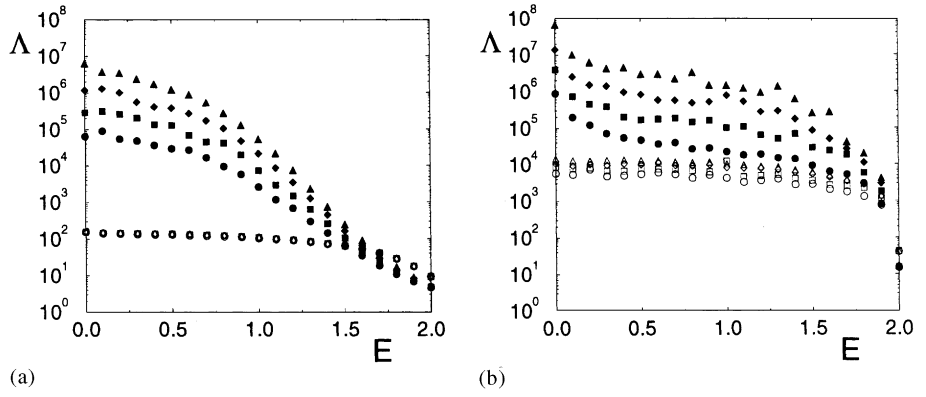

Fig. 1. $\Lambda(E)$ for correlated chains with $\gamma = 0.1$ (full symbols) of disorder amplitude (a) $\lambda = 1.0$ and (b) $\lambda = 0.1$. For comparison, the results for uncorrelated chains with the same variance of the potentials are also shown (open symbols). The system sizes were $N = 2^{n_0}$ with $n_0 = 13(\circ), 15(\square), 17(\diamond)$ and $19(\triangle)$.

$[-1, 1]$, with $\langle V \rangle = 0$, and λ is a positive constant describing the amplitude of the disorder. In contrast to the conventional Anderson model, where the V_n are uncorrelated, we consider site energies that are long-range correlated with the correlation function $C(\ell)$, decaying by a power law [6],

$$C(\ell) \equiv \langle V_n V_{n+\ell} \rangle \sim \frac{1}{N} \sum_{n=1}^{N} V_n V_{n+\ell} \sim (1 + \ell^2)^{-\gamma/2} \to \ell^{-\gamma}, \quad 0 < \gamma < 1. \tag{2}$$

For solving Eq. (1) numerically, we generated correlated one-dimensional systems with up to 2^{19} sites, using the method of double Fourier transform [6]. We computed the localization lengths $\Lambda(E, \lambda)$, following the transfer-matrix method [7,8]. Averages were taken over 100 chains. Fig. 1 shows $\Lambda(E)$ for $|E| \in [0; 2]$ for correlated chains of correlation exponent $\gamma = 0.1$ with (a) $\lambda = 1.0$ and (b) $\lambda = 0.1$. For comparison, the results for uncorrelated chains with the same variance of the potentials are also shown. Close to the band edge the states of the correlated systems are also localized with Λ even smaller than in the uncorrelated case. This is a surprising result indicating that the presence of correlations makes the states more strongly localized. A scaling theory has been developed to explain this regime [9].

Close to the band center, however, the localization lengths Λ of the correlated systems rise to very high values and scale roughly linearly with the system size. This seems to indicate extended states, but localized states with localization lengths larger than the considered system sizes cannot be excluded. In order to decide between these two possibilities we have applied the method of level statistics.

Level statistics is a powerful tool for determining the localization properties of electronic wave functions [10,11]. For extended states, the level spacing distribution $P(s)$ of consecutive eigenvalues (levels) E_i shows the universal random matrix theory result, which is well approximated by the Wigner surmise, $P(s) = (\pi/2)s \exp(-\pi s^2/4)$. Here, $s = |E_i - E_{i-1}|/\Delta$ where Δ is the mean level spacing in the energy interval

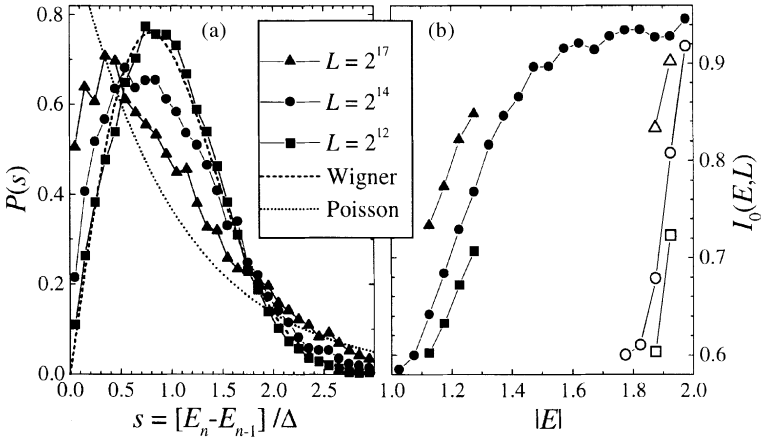

Fig. 2. (a) Level spacing distribution $P(s)$ for the one-dimensional Anderson model with correlated diagonal disorder $\gamma = 0.1$ and $\lambda = 1$ in the energy interval $1.15 < E < 1.20$. The data for three system sizes is compared to the Wigner and the Poisson distribution. (b) Energy dependence of $I_0 = \langle s^2 \rangle / 2$ for the same three system sizes and $\lambda = 1$ (full symbols) and $\lambda = 0.1$ (open symbols). Since $I_0(E, L)$ increases with increasing system size, all states are localized.

considered. For localized states the uncorrelated eigenvalues are described by the Poisson distribution, $P(s) = \exp(-s)$. For finite systems, the shape of $P(s)$ is in between the two limiting cases and approaches one of them with increasing system size (see also [12]).

Fig. 2a shows the system-size dependence of the $P(s)$ distribution for $\gamma = 0.1$ in a representative energy interval. The eigenvalues for the distributions were numerically calculated using a Lanczos method. Evidently, the distribution approaches the Poisson limit with increasing system size, even though $P(s)$ is approximately equal to the Wigner limit for the smallest system size considered.

Since it is tedious to compare the $P(s)$ distributions for several system sizes and energy intervals, we calculate the size dependence of the quantity $I_0 = \langle s^2 \rangle / 2$, which is related to the second moment of the level spacing distribution $\langle s^2 \rangle = \int_0^\infty s^2 P(s)\, \mathrm{d}s$. If the eigenstates are localized, I_0 increases monotonically with increasing system size approaching the Poisson limit $I_0^{\mathrm{loc}} = 1$ for infinite system size. If the modes are extended, I_0 decreases monotonically and reaches the Wigner limit $I_0^{\mathrm{ext}} \approx 0.6$. Directly at a transition from localized to extended states, the states are "critical" and the level spacing distribution $P(s)$ as well as the quantity $I_0 = I_0^{\mathrm{crit}}$ are system-size independent. Such a transition has been found e.g. for the uncorrelated Anderson model in $d = 3$ [10,11], but not in $d = 2$ [13]. The value of I_0^{crit} is not universal, but it is always clearly above $I_0^{\mathrm{ext}} \approx 0.6$.

Fig. 2b shows our results for $I_0(E, L)$ for three system sizes L and two values of the disorder width λ. First, we discuss the case $\lambda = 1$. For $1.1 < |E| < 1.3$, the quantity $I_0(E, L)$ increases with increasing L and all states are localized in this energy regime. For $|E| > 1.3$, $I_0(E, 2^{14})$ is even closer to $I_0^{\mathrm{loc}} = 1$, indicating more strongly localized

states. For $|E| < 1.1$, $I_0(E, 2^{14})$ is near to the Wigner limit $I_0^{\text{ext}} \approx 0.6$. At the first glance, this could be interpreted as an indication for extended states at small energies $|E|$. But this interpretation cannot be correct: If the wave functions became extended for some small $|E|$, there would be a transition from localized to extended states somewhere in the spectrum. At such a transition $I_0(E, L)$ has to be system-size independent *and* equal to I_0^{crit}. Both conditions cannot be fulfilled (for the same energy E), since $I_0(E, 2^{14})$ is smaller than $I_0(E, 2^{17})$ for $|E| > 1.1$ *and* smaller than any possible value of I_0^{crit} for $|E| < 1.1$. Thus, there cannot be any transition from localized to extended states, and all states must be localized including those for $|E| < 1.1$. Hence, the small values of $I_0(E)$ for $|E| < 1.1$ are not due to extended states but to weakly localized states with localization lengths much larger than the system sizes L. The same kind of behavior has been observed for the uncorrelated Anderson model with diagonal disorder in $d = 2$, where also all wave functions are localized [13].

We find the same type of behavior for other values of the disorder width λ and for other correlation exponents γ with $0 < \gamma < 1$. Our results for $\lambda = 0.1$ are included in Fig. 2b for the energy range $1.8 < |E| < 2.0$. The same arguments as in the previous paragraph apply here.

We have strong evidence from level statistics that the wave functions in the band center are not extended but weakly localized modes with finite localization lengths much larger than the system sizes. We also want to point out that in this case the direct calculation of the localization length, using the transfer-matrix method, is not sufficient to decide about the question if states are extended or not.

Note added in proof

After completing this work, we learnt of recent articles by Moura and Lyra [14], where Anderson chains with correlated site energies have been studied using a renormalization group technique. The conclusions of Moura et al. for correlation exponents $\gamma > 0$ agree with our findings.

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